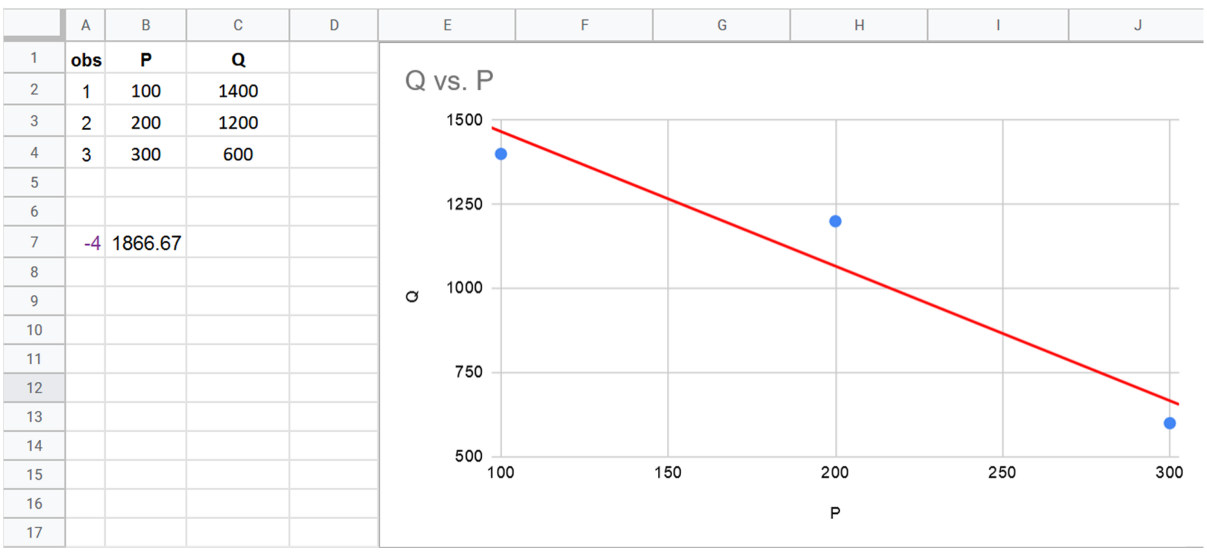
Problem 1

Estimate the demand function for the data below using a spreadsheet.

	A	B	C
1	**obs**	**P**	**Q**
2	1	100	1400
3	2	200	1200
4	3	300	600

Solution 1

What we want to do is estimate the line of best fit through the data (shown in red below), making sure that Q is on the vertical axis and P is on the horizontal axis. *You will not get the same result if you mix up the axes.*



The relevant command in either Google sheets or Excel is `linest()`, which will give you the slope and the intercept of the line. In this case, we would use

```
=linest(C2:C4,B2:B4)
```

Notice that the first argument `C2:C4` captures data for Q and the second argument `B2:B4` captures data for P, *and it has to be in this order.*

After entering the command into a cell, you'll see the numbers `-4 1866.67`. The number 1866.67 is the vertical intercept of the demand curve and -4 is the slope; therefore the estimated demand curve is

$$Q = 1866.67 - 4P.$$

Solve the demand curve for P to find the inverse demand curve,

$$P = 466.67 - 0.25Q.$$

In the homework, you would then use the inverse demand function to solve the remainder of the problem. And one last time: *yes, you really do have to estimate the demand function first and then invert the estimated demand function.* If you don't believe me, then try estimating the inverse demand function directly; you'll get different (i.e. wrong) numbers.

Problem 2

Consider the following spatial competition problem.

- There are two pizza places, each on the opposite end of a mile-long street evenly filled with consumers. Both pizza places are delivery-only.
- The pizza place on the far-left end of the street—Pizza Place A—sells a pizza that consumers think has a value of $v_A = 22$. Their marginal cost per pizza is $c_A = 1$.
- The pizza place on the far-right end of the street—Pizza Place B—sells a pizza that consumers think has a value of $v_B = 25$. Their marginal cost per pizza is $c_B = 2.5$.
- Each pizza place charges $a = 2$ per mile for delivery.

Complete the following:

(a) Find the equilibrium price for each pizza place.

Solution. Utility for going to Pizza Place A is $U_A = 22 - P_A - 2d$, and utility for going to Pizza Place B is $U_B = 25 - P_B - 2(1-d)$. The indifferent consumer satisfies

$$22 - P_A - 2d = 25 - P_B - 2(1-d) \quad \Longrightarrow \quad d^* = \frac{P_B - P_A - 1}{4},$$

from which we conclude that

$$Q_A = \frac{P_B - P_A - 1}{4}, \quad Q_B = 1 - \frac{P_B - P_A - 1}{4} = \frac{P_A - P_B + 5}{4}.$$

It follows that Pizza Place A has profit function of

$$\begin{aligned}
\Pi_A &= P_A Q_A - c_A Q_A \\
&= P_A \left(\frac{P_B - P_A - 1}{4} \right) - 1 \left(\frac{P_B - P_A - 1}{4} \right) \\
&= \left(\frac{P_A P_B - P_A^2 - P_A}{4} \right) - 1 \left(\frac{P_B - P_A - 1}{4} \right).
\end{aligned}$$

Differentiate Π_A with respect to P_A and set it equal to zero and you get

$$\left(\frac{P_B - 2P_A - 1}{4}\right) - 1\left(\frac{-1}{4}\right) := 0 \quad \implies \quad P_A = \frac{P_B}{2}.$$

Likewise, Pizza Place B has profit function of

$$\begin{aligned}
\Pi_B &= P_B Q_B - c_B Q_B \\
&= P_B\left(\frac{P_A - P_B + 5}{4}\right) - 2.5\left(\frac{P_A - P_B + 5}{4}\right) \\
&= \left(\frac{P_A P_B - P_B^2 + 5P_B}{4}\right) - 2.5\left(\frac{P_A - P_B + 5}{4}\right).
\end{aligned}$$

Differentiate Π_B with respect to P_B and set equal to zero and you get

$$\left(\frac{P_A - 2P_B + 5}{4}\right) - 2.5\left(\frac{-1}{4}\right) := 0 \quad \implies \quad P_B = \frac{P_A + 7.5}{2}.$$

We now have two equations for two unknowns. I'll use substitution to solve.

$$\begin{aligned}
P_B &= \frac{P_A + 7.5}{2} \\
&= \frac{\frac{P_B}{2} + 7.5}{2} \\
\implies \quad 2P_B &= \frac{P_B}{2} + 7.5 \\
\implies \quad 4P_B &= P_B + 15 \\
\implies \quad P_B^* &= \mathbf{5}
\end{aligned}$$

Now plug the solution to P_B into the BRF for P_A and you get $P_A^* = \mathbf{2.5}$.

(b) Find the equilibrium quantity sold for each pizza place.

Solution. Plug in the solutions for P_A and P_B to get

$$\begin{aligned}
Q_A &= \frac{P_B - P_A - 1}{4} = \frac{5 - 2.5 - 1}{4} = \mathbf{0.375} \\
Q_B &= \frac{P_A - P_B + 5}{4} = \frac{2.5 - 5 + 5}{4} = \mathbf{0.625}.
\end{aligned}$$

(c) Find the equilibrium profit for each pizza place.

Solution. Plug in the solutions for P_A, Q_A, P_B, and Q_B to get

$$\Pi_A = P_A Q_A - c_A Q_A = (2.5)(0.375) - (1)(0.375) = \mathbf{0.5625}$$
$$\Pi_B = P_B Q_B - c_B Q_B = (5)(0.625) - (2.5)(0.625) = \mathbf{1.5625}$$