Problem 1

A grocery store sells vegan cookies. Vegans have high valuation for vegan cookies, but non-vegans low valuation for vegan cookies. Specifically, the grocery store can sell a bag with 20 cookies and/or a bag with 10 cookies. One bag per customer. Non-vegans don't want 20 cookies, but vegans are interested in both the 10- and 20-cookie bag.

Let P_X denote the price of a bag of X cookies. (For example, P_{10} is the price of a 10-cookie bag.) A vegan has a cookie utility function of

$$U_H(X) = 8X - P_X, \quad X \in \{10, 20\},$$

and a non-vegan has a cookie utility function of

$$U_L(X) = 6X - P_X, \quad X \in \{10\}.$$

The proportion of vegans is α and the marginal cost of a vegan cookie is 4. Finally, suppose the store can prevent resale. (Perhaps they force you to eat them all right then and there because they are super-exclusive cookies and they don't want their secrets leaked.)

(a) Explain why the grocery store cannot engage in third-degree price discrimination.

Solution. Vegans don't walk around wearing a shirt that says "LOOK AT ME I'M A VEGAN" on it, so the grocery store can't observe who vegans are. (Okay, I guess some do. But let's suppose otherwise.) A vegan could just lie and say they were non-vegan and buy the cookies at the lower non-vegan price. Therefore the grocery store can't effectively set a separate price for vegans and non-vegans.

(b) Express profit as a function of α if the grocery store sells only 20-cookie bags.

Solution. Vegans will only buy a 20-cookie bag at price P_{20} if

$$U_H(20) \geq 0 \quad \iff \quad 8(20) - P_{20} \geq 0 \quad \iff \quad P_{20} \leq 160.$$

The grocery store will charge the highest price it can actually get away with, $P_{20}^* = 160$. Non-vegans don't want 20 vegan cookies so they don't buy anything.

The number of vegans is α, each of whom purchase a bag at a price of 160, which amounts to total revenue of TR $= 160\alpha$. The marginal cost per cookie is 4, there are 20 cookies per bag, and α bags are sold, giving total cost of TC$= 80\alpha$. So profit is

$$\Pi^* = 160\alpha - 80\alpha = \mathbf{80\boldsymbol{\alpha}}.$$

(c) Express profit as a function of α if the grocery store decides to sell 10-cookie bags to non-vegans and 20-cookie bags to vegans.

Solution. Non-vegans only consider buying the 10-cookie bag, which they do at price P_{10} when

$$U_L(10) \geq 0 \iff 6(10) - P_{10} \geq 0 \iff P_{10} \leq 60.$$

So the grocery store will charge the highest price it can to get non-vegans on board with the 10-cookie bag, $P_{10}^* = \mathbf{60}$.

Vegans have a choice: buy a 10-cookie bag for $P_{10}^* = 60$ or a 20-cookie bag for P_{20}. If they purchase a 10-cookie bag, then their utility is

$$U_H(10) = 8(10) - 60 = 20.$$

Or vegans could buy a 20-cookie bag at price P_{20}, which gives utility

$$U_H(20) = 8(20) - P_{20} = 160 - P_{20}.$$

Vegans prefer buying the 20-cookie bag when

$$U_H(20) \geq U_H(10) \iff 160 - P_{20} \geq 20 \iff P_{20} \leq 140.$$

If $P_{20} > 80 + P_{10}$, then vegans would rather just buy the 10-cookie bag because the price is too high to warrant buying the 20-cookie bag. So the grocery store will charge the highest price it can to get vegans on board with the 20-cookie bag, $P_{20}^* = \mathbf{140}$.

Let's piece it all together now. For vegans:

- The number of vegans is α, each of whom buy a bag at price 140, giving vegan revenue of $\text{TR}' = 140\alpha$.
- Each cookie has a marginal cost of 4, so each big bag has a marginal cost of $20 \times 4 = 80$, so the total cost is $\text{TC}' = 80\alpha$ because α bags are sold.
- Therefore profit from vegans is $\Pi' = 140\alpha - 80\alpha = \boldsymbol{60\alpha}$.

And now for non-vegans:

- The number of non-vegans is $1 - \alpha$, each of whom buy a bag at price 60, giving non-vegan revenue of $\text{TR}'' = 60(1 - \alpha)$.
- Each cookie has a marginal cost of 4, so each small bag has a marginal cost of $10 \times 4 = 40$, so the total cost is $\text{TC}'' = 40(1 - \alpha)$ since $1 - \alpha$ bags are sold.
- Therefore profit from non-vegans is $\Pi'' = 60(1 - \alpha) - 40(1 - \alpha) = \boldsymbol{20(1 - \alpha)}$.

Total profit is given by adding the profit from vegans and the profit from non-vegans,

$$\Pi_{H+L}^* = 60\alpha + 20(1 - \alpha) = \boldsymbol{20 + 40\alpha}.$$

(d) Determine for which values of α the grocery store should target only vegans, and for which the grocery store should target both types of buyers.

Solution. The grocery store will target both types when it is more profitable to do so, that is, when

$$\Pi^*_{H+L} \geq \Pi \quad \iff \quad 20 + 40\alpha \geq 80\alpha \quad \iff \quad \boldsymbol{\alpha \leq \frac{1}{2}}.$$

The intuition is that the grocery store also targets non-vegans when there are enough non-vegans (small α), but only targets vegans when there are not many non-vegans (large α).

One last comment. Suppose there *is* resale and $\alpha \leq 1/2$. Then two non-vegans could get together, purchase two 10-cookie bags for a total of \$120, and sell 20 cookies to vegans for, say, \$130. The two non-vegans benefit because they're profiting \$5 each. The vegan benefits because they're paying only \$130 for 20 cookies. Point is, preventing resale is also necessary for second-degree price discrimination even with incentive-compatible pricing.

Problem 2

(a) A firm selling a good to consumers in two cities should set a higher markup in the city in which demand for the product is elastic.

False. Recall the Lerner Index, which says

$$\frac{P - \text{MC}}{P} = -\frac{1}{\epsilon_D},$$

that is, the profit-maximizing percentage markup is the negative inverse of demand elasticity. The city with more elastic demand has a larger ϵ_D, which is in the denominator, which makes $-1/\epsilon_D$ relatively small, therefore implying a lower markup.

(b) Two firms are geographically differentiated (e.g., gas stations on opposite sides of Davis). If travel costs decline, we should expect the cross-price elasticities of the two products to become more positive.

True. If travel costs decline, then a consumer's decision is more heavily determined by which firm has the lower price: my competitor raises their price, so tons of consumers come to me instead, and my quantity sold increases dramatically (i.e. very positive cross-price elasticity).

In fact if travel costs are zero, then the game turns into pure Bertrand competition in which cross-price elasticities are infinite.

(c) A monopoly that can engage in first-degree price discrimination sells more units of a good than a monopoly that cannot price discriminate.

True. A monopolist that cannot price discriminate restricts production in order to set a price above marginal cost. A monopolist that can engage in first-degree price discrimination charges a specific price to each consumer based on each consumer's valuation of the good, which means every consumer with a valuation above marginal cost will be a sale.

(d) Relative to a monopoly that can't price discriminate, first-degree price discrimination lowers welfare.

False. Quantity supplied is restricted without price discrimination, which leads to a deadweight loss. But with first-degree price discrimination, quantity will be supplied to every consumer with valuation above marginal cost, therefore implying zero deadweight loss. (Of course, this means consumer surplus is zero.)

(e) Relative to the case in which a firm must charge the same price to everyone, third-degree price discrimination makes the firm better off, but all consumers worse off.

False. Firms are better off since they can set different prices that more closely reflect the valuation and elasticity of each group, i.e. can set prices that squeezes more profit out of each group. This means charging a higher price to high-valuation customers and a lower price to low-valuation customers. High-valuation customers hate this. But low-valuation customers like it: they might have not even been purchasing before because the single price was too high; and if they were customers before, then they're happier now because they're paying a lower price.

(f) The Nash equilibrium of a prisoner's dilemma has the highest combined payoffs.

False. The defining characteristic of a prisoner's dilemma is that each player receives a smaller payoff than they would have received had they cooperated; and therefore the combined payoff is also smaller.

Think back to question 1 of homework 1. When there are two firms Cournot competing, each firm produces 300 and receives profit of \$1,800,000, so the combined profit of each firm is \$3,600,000. On the other hand, the two firms could do better if they colluded, implicitly functioning as a monopoly to limit quantity to $450/2 = 225$ each and splitting $\$4,050,000/2 > \$1,800,000$.

Problem is, each firm has an incentive to cheat on any cooperative agreement they make. Firm A's best response function is $Q_A = (900 - Q_B)/2$, so if Firm B sticks with the agreement and produces $Q_B = 225$, then Firm A's profit-maximizing response is to produce

$$Q_A^* = (900 - 225)/2 = 337.5,$$

illustrating that Firm A's best response is to cheat on the agreement and instead produce more than 225. The same logic holds for Firm B. Therefore it cannot be an equilibrium to collude and act as a monopoly, so instead they compete and take the smaller profit of \$1,800,000 each; such is the prisoner's dilemma.