Problem 1. A firm distributes natural gas to a town with 100 households. The firm has fixed costs of $F C=10,000$, marginal cost of $M C=10$, and the inverse demand curve for each household is $P=30-Q$.
(a) Explain why we know this firm is a natural monopoly.

Solution. The firm has positive fixed costs and constant marginal cost, which always implies a natural monopoly. Total cost is $\mathrm{TC}=\mathrm{FC}+\mathrm{MC} \times Q$. Divide both sides by $Q$ and you get the average total cost $\mathrm{AC}=\mathrm{FC} / Q+\mathrm{MC}$. Therefore average cost is always decreasing in $Q$. It follows that the demand curve intersects the AC curve at a downward-sloping part of the AC curve (because every part of the AC curve is downward-sloping).
(b) Explain why if the regulator sets a linear price of $P=20$, then the firm's economic profits will be equal to zero. Also find consumer surplus.
Solution. When $P=20$, each household demands $Q=10$ units of the good. Since there are 100 households, total quantity demanded in the market is $Q_{M}=1,000$. Therefore the firm's profit will be

$$
\begin{aligned}
\Pi & =(P-\mathrm{MC}) Q_{M}-\mathrm{FC} \\
& =(20-10) 1,000-10,000 \\
& =0
\end{aligned}
$$

Alternatively, profit is zero when $P=A C$. Average cost is $10,000 / 1,000+10=20$.
Each household has a surplus of $C S=0.5(30-20) 10=50$, and there are 100 households, so $\mathrm{CS}_{M}=5,000$.
(c) You convince the regulator to use fixed-price (a.k.a price cap) regulation: the price will stay at $P=20$ even if the firm finds ways to lower its costs. The firm can lower its marginal cost by exerting effort according to $\mathrm{MC}(e)=10-e$, but exerting effort costs $C(e)=100 e^{2}$ because a bonus is paid to employees who work harder.
Will the firm invest to lower its marginal cost? If so, how much profit will the firm earn under fixed-price regulation?
Solution. The regulator has fixed $P=20$ and therefore household quantity is fixed at $Q=10$ and market quantity is fixed at $Q_{M}=1,000$. Firm profit is therefore

$$
\begin{aligned}
\Pi & =(20)(1,000)-(10-e)(1,000)-100 e^{2}-10,000 \\
& =1,000 e-100 e^{2}
\end{aligned}
$$

Notice that we get the same profit as in part (b) when $e=0$.

Anyway, take the derivative with respect to $e$ and set it equal to zero, which gives

$$
\frac{d \Pi}{d e}=1,000-200 e:=0 \quad \Longrightarrow \quad e^{*}=5
$$

The firm will exert enough afford to drive its marginal cost down to $10-5=5$; exerting that much effort costs $100(5)^{2}=2500$; but it's worth it because doing so increases profit to $1,000(5)-100(5)^{2}=2500$, which is definitely more than zero.
(d) Suppose the regulator can identify two different types of households: one type with low elasticity and the other with high elasticity. In general terms, how would the regulator's pricing decision change relative to part (b)? How would this affect consumer surplus?
Solution. The regulator could charge the inelastic types a higher price $P_{H}>20$ and the elastic types a lower price $P_{L}<20$, while still ensuring that the firm earned zero economic profit; this is called Ramsey pricing. The result would be a slightly lower consumer surplus for the inelastic types, but a much larger consumer surplus for the elastic types, so consumer surplus (and therefore total surplus) increases overall.
(e) What is the optimal two-part tariff that the regulator could set if it wanted to maximize welfare?

Solution. The optimal two-part tariff consists of, um, two parts. For the first part, set $P=\mathrm{MC}$, which ensures zero deadweight loss. Ergo $P=10$. For the second part, divide up the fixed cost among all 100 households, i.e. tax each household $T=10,000 / 100=100$ and give it to the firm so that the firm earns zero profit.
(f) How much would consumer and producer surplus change if the regulator set the optimal two-part tariff instead of the linear price of $P=20$ ?
Solution. Producer surplus is unchanged because it's deliberately set to zero whether we're talking about linear prices, Ramsey prices, or a two-part tariff.
With the two-part tariff, $P=10$ so $Q=20$ for each household, ergo each household gets pre-tax surplus of $0.5(30-10) 20=200$. But each household pays the tax of 100, so after-tax surplus for each household is $C S=100$. There are 100 such households, so consumer surplus will aggregate to $\mathrm{CS}_{M}=10,000$.

Each household surplus increases by 50 units and total consumer surplus by 5,000. Let us conclude, and forget about exerting effort for a moment. Since profit is deliberately set to zero in all cases, welfare is driven entirely by consumer surplus. Welfare is therefore highest with a two-part tariff, and lowest with a linear price. Note that Ramsey pricing, in which a higher price is set for inelastic consumers and a lower price is set for elastic consumers, is somewhere in between. That is,

$$
\text { linear pricing welfare }<\text { Ramsey pricing welfare }<\text { two-part tariff welfare. }
$$

Calculating Ramsey prices is hard so we won't do it, but you should be familiar with its meaning, implications, limitations, etc.

Problem 2. State whether the following statements are true or false and explain why.
(a) If unregulated, a monopoly using Mickey Mouse pricing generates higher profits and greater deadweight loss than a monopoly that uses linear pricing.
False. Mickey Mouse pricing is just a two-part tariff set by a monopolist absent any regulation. The monopolist sets the price per unit at $P=\mathrm{MC}$, which ensures zero deadweight loss. This is like charging $P=\mathrm{MC}$ every time someone wants to go on a ride at Disneyland.

But the monopoly also charges a fixed cost to everyone who wants access to the good in the first place, which extracts all consumer surplus. This is like charging for a pass to enter Disneyland, whether a person goes on any rides or not.
(b) Ramsey prices maximize a firm's profits by setting high prices for inelastic customers and low prices for elastic customers.
False. Ramsey prices set higher prices for inelastic customers and low prices for elastic customers. But the point is not to maximize a firm's profit - it is to maximize welfare (i.e. minimize deadweight loss). In fact, these prices are set by a regulator to ensure that a firm's economics profits are equal to zero.
(c) Cost-plus regulation provides better incentives for the firm to offer good service than fixed-price regulation.
True. Fixed-price regulation means the regulator tells the monopoly that they can sell their good for the foreseeable future at some constant price $P$. If the monopolist is able to become more efficient and reduce its marginal cost, then it is therefore allowed to earn positive profit: it has an incentive to become more efficient.
But under cost-plus regulation, the regulator will reduce the price if marginal cost falls, which means the monopoly won't be able to benefit from being more efficient, and therefore won't bother trying to be more efficient.
(d) If an incumbent sets prices below an entrant's marginal cost, then the incumbent is guilty of predatory pricing.
False. If an incumbent sets its price below its own marginal cost, then things are fishy: it's difficult to imagine a scenario in which the firm would accept negative profit unless they were trying to drive a weaker competitor out of the market and then afterwards going to monopoly profit.
But maybe an incumbent has been in the industry for a long time and has become very efficient, which leads to lower price; whereas an entrant might be less efficient and therefore has high marginal costs that the incumbent incidentally undercuts.

Remember, there is a conduct requirement that must be satisfied to qualify as anticompetitive behavior: "The violator must engage in activity which a normal firm competing on the basis of cost and / or quality would not."
(e) If an incumbent with a dominant market position bundles goods and drives an entrant out of the market, then bundling is anti-competitive.
False. There are reasons why a firm might choose to bundle independent of competitive reasons. Bundling can be convenient for consumers: "batteries not included" is annoying. Bundling can expose consumers to new products: you might subscribe to Netflix because you want to watch the series Stranger Things, and end up also watching something you never expected to watch like Tiger King. Bundling can allow for close integration between the products: Microsoft Office products work really well together.

Or maybe the entrant just kinda sucked and was going to exit anyway.
Point is, there are a lot of factors that need to be examined beyond merely whether an entrant stayed in the market or not. Again, note the conduct requirement: "The violator must engage in activity which a normal firm competing on the basis of cost and/or quality would not," and bundling can absolutely be "normal" behavior.

