CAPM and Portfolio Beta

Each asset has its own beta. The beta of a portfolio, then, is a convex combination of the betas of each asset that constitute the portfolio.

For example, if your portfolio is 1/3 equity that has β_E, and 2/3 bonds that have β_D, then your portfolio has beta

$$\beta_P = \frac{1}{3} \times \beta_E + \frac{2}{3} \times \beta_D.$$

In generality, suppose the portfolio has $i = 1, ..., n$ assets with respective betas β_i and respective weights α_i. Then the portfolio beta is

$$\beta_P = \sum_{i=1}^{n} \alpha_i \beta_i.$$

Perfect Capital Markets Assumption

More CAPM

In order for price to equal present value, we need perfect capital markets, in addition to either

- a non-trivial mass of risk neutral traders (on both sides of the market), or
- enough heterogeneity in risk preferences so it is as if there is a representative risk-neutral agent.

Or to more bluntly capture the two bullet points: there needs to be *some* hypothesis that says that there is *some* risk-neutrality in the aggregate.

Equity Premium Puzzle

Rates of equity premia have been persistently higher than the risk-free rate. But this goes against theory, which predicts that a high equity premium would induce people to buy more stocks, which would in turn drive up the price of stocks, and hence reduce their returns until the equity premium shrunk to zero.

Adding risk aversion can't explain the puzzle – we will consider weakening the perfect capital markets assumption.

(Im)Perfect Capital Markets

Perfect capital markets rely upon four assumptions.

(a) *No Differences in Opinion.* Everyone agrees on probabilities in cases of uncertainty.

(b) *Infinite Depth.* There are effectively infinitely many buyers and sellers (no individual decision can affect the market).

(c) *No Transaction Costs.* Buying and selling is free, i.e. no brokerage fees.

(d) *No Taxes.* Special focus on capital gains tax: whoever sells an asset is not subject to their return being taxed. This is a biggie.

Taxes

Taxes affect the after-tax net present value of a project. They also create a wedge between borrowing and lending rates – lenders see their returns taxed and thus their after-tax interest rate is lower than that of borrowers.

A **capital gains tax** enacts a tax on any increase in the price of an asset when you sell it. For example, if you buy a stock for \$100 and sell it a year later for \$110, then your capital gain is \$10. If the capital gains tax is 10%, then you will only get to keep \$9 of that capital gain.

The practical takeaways are as follows.

- Post tax-rate is the relevant return-rate on a bond.
- The appropriate opportunity cost of capital is the highest post-tax lending rate obtainable.
- Use the lending rate to express future cash flows in present day dollars.

Modigliani-Miller Theorems

These theorems address the problem of *optimal capital structure*, that is, whether a manager should obtain funding via debt, equity, or a mixture, bearing in mind that debt holders must be paid back before equity.

Modigliani-Miller Theorem 1

Theorem 1 (Modigliani-Miller Theorem 1)**.** *In a perfect capital market, the total value of a firm is equal to the market value of the total cash flows generated by its assets, and is not affected by its choice of capital structure.*

The present value of a project must equal the present value of the firm's issued claims today (i.e. equity plus debt).

A 100% equity firm must have equity price equal to project PV. If the firm finances with 50% debt + 50% equity, then the combined price of the two must sell *for the same PV*.

In other words, it doesn't matter how the firm funds the project – the capital structure cannot change the project PV. Sometimes this is called the *irrelevance theorem*.

In the maths we can write

$$D + E = A,$$

where D is the price of debt; E is price of (levered) equity; and $A = \text{PV}$ is the present value of the project.

Modigliani-Miller Theorem 2

Theorem 2 (Modigliani-Miller Theorem 2)**.** *The cost of capital of levered equity, r_E, increases with the firm's market value debt-equity ratio. Specifically,*

$$r_E = r_0 + \frac{D}{E}(r_0 - r_D),$$

where r_0 is the cost of capital for a firm with no debt; r_D is the cost of capital for a firm with some debt; D is the amount borrowed; and E is the amount of equity.

The intuition is that since the firm has to pay back debt before anything else, a large debt increases the risk that the firm won't have much left over for shareholders – equity becomes riskier. So large debt D relative to equity E increases r_E to compensate for that risk.