Problem 1

Treat Brazil (currency *real*, code BRL) as the *home* country. Suppose the cost of the market basket in the United States is $P_{US} = 190\,\text{USD}$, the exchange rate is 4.07 BRL per 1 USD, and the price of a market basket in Brazil is 520 BRL.

(a) Determine the price of a US basket in BRL.

Solution. Just use the typical dimensional analysis tricks, which gives

$$190\,\cancel{\text{USD}} \times \frac{4.07\,\text{BRL}}{1\,\cancel{\text{USD}}} = 773.3\,\text{BRL}\,.$$

(b) Determine whether or not PPP holds.

Solution. The real exchange rate is

$$\begin{aligned} q_{\text{BRL / USD}} &= \frac{\left(\frac{773.3\,\text{BRL}}{1\text{ US basket}}\right)}{\left(\frac{520\,\text{BRL}}{1\text{ Brazilian basket}}\right)} \\ &= \frac{773.3\,\cancel{\text{BRL}}}{1\text{ US basket}} \frac{1\text{ Brazilian basket}}{520\,\cancel{\text{BRL}}} \\ &= \frac{1.49\text{ Brazilian basket}}{1\text{ US basket}}, \end{aligned}$$

although it would typically be expressed just as 1.49. Note that the currency used for denomination will not affect the real exchange rate; we'd just be multiplying both numerator and denominator by the same number if we converted to USD. Just make sure the home price goes in the denominator.

Anyway, PPP does not hold: the prices of the baskets (after currency conversion) should be identical, and therefore the real exchange rate should be 1, if PPP holds.

(c) Determine whether the real overvalued or undervalued.

Solution. The real is undervalued against the USD.

Note that, simply as a matter of terminology, the real exchange rate tells us how overvalued or undervalued the *foreign* currency is. In this case, the foreign currency is the USD, so we say that the USD is 49% overvalued against the real.

It does follow that the BRL is undervalued against the USD, but not by 49% as you might immediately think. To see why, flip everything around and now think of the US as being the home country, Brazil as the foreign country. Then the real exchange rate is $q_{\text{USD / BRL}} = 520/773.3 = 0.672$, implying that the BRL is undervalued against the USD by 32.8%.

(d) Determine whether the real is expected to appreciate or depreciate.

Solution. You might immediately think that an undervalued currency will appreciate over time. You would be correct.

We'd expect the real exchange rate to gravitate back to 1 over time, which means we expect the real to appreciate against the US dollar: the overvalued currency depreciates and the undervalued currency appreciates.

Let's break that down. Right now, we don't have PPP because

$$\underbrace{190\,\text{USD} \times E_{\text{BRL / USD}}}_{773.3\,\text{BRL}} > 520\,\text{BRL}\,.$$

Supposing domestic prices are fixed, this only becomes an equality $E_{\text{BRL / USD}}$ becomes less than 4.07, that is, an appreciation of the BRL. Indeed, note that the exchange rate satisfying PPP is

$$190\,\text{USD} \times E_{\text{BRL / USD}} = 520\,\text{BRL} \quad \Longrightarrow \quad E_{\text{BRL / USD}} = 2.74.$$

In other words, the exchange rate has to change by a proportion of

$$\frac{2.74 - 4.07}{4.07} = -0.3268,$$

that is, appreciate by 32.68%, for PPP to hold.

Problem 2

For the US and Europe, suppose that inflation forecasts are $\pi^e_{\text{USD}} = 3\%$ and $\pi^e_{\text{EUR}} = 1\%$, respectively. Consider the following scenarios concerning one year later.

(a) If absolute PPP holds, then what would we forecast for the depreciation in USD relative to the EUR by next year?

Solution. The real exchange rate is defined to be

$$q_{\text{USD / EUR}} \equiv \frac{P_{\text{EUR}} \times E_{\text{USD / EUR}}}{P_{\text{USD}}}.$$

Solving for the nominal exchange rate gives

$$E_{\text{USD / EUR}} = q_{\text{USD / EUR}} \times \frac{P_{\text{USD}}}{P_{\text{EUR}}}.$$

In terms of percentages, the preceding equation can be translated into rates of change with an approximation of

$$\%\Delta E_{\text{USD / EUR}} = \%\Delta q_{\text{USD / EUR}} + \pi_{\text{USD}} - \pi_{\text{EUR}},$$

or in words: the rate of nominal depreciation equals the rate of real depreciation plus the inflation differential.

If PPP holds, then $q_{\text{USD / EUR}} = 1$ always, and therefore its rate of change is zero. It follows that the rate of expected depreciation is forecast as

$$\%\Delta E^e_{\text{USD / EUR}} = 0 + 3\% - 1\% = 2\%.$$

(b) Suppose PPP fails: a US basket of a good costs 100 USD, whereas the euro cost of the same basket – after accounting for the exchange rate – is 130 USD. What would we forecast for the depreciation in USD relative to the EUR by next year?

Solution. Let's use the starting point of

$$\%\Delta E_{\text{USD / EUR}} = \%\Delta q_{\text{USD / EUR}} + \pi_{\text{USD}} - \pi_{\text{EUR}},$$

and note that deviations from $q_{\text{USD / EUR}} = 1$ decay at a rate of roughly 15 percent per year. Initially the real exchange rate is

$$q_{\text{USD / EUR}} = \frac{130}{100} = 1.3,$$

so the PPP deviation is 0.3.

One year later, that PPP deviation of 0.3 is expected to decay by 15 percent and therefore is expected to be only $0.85 \times 0.3 = 0.255$. It follows that the new real exchange rate is expected to be 1.255. Therefore the expected rate of change in the real exchange rate is

$$\%\Delta q^e_{\text{USD / EUR}} = \frac{1.255 - 1.3}{1.3} = -3.46\%.$$

Now we have everything we need to forecast the change in the nominal exchange rate, namely,

$$\%\Delta E^e_{\text{USD / EUR}} = -3.46\% + 3\% - 1\% = -1.46\%.$$

The dollar is expected to appreciate, which makes sense on an intuitive level: the USD is initially undervalued, so the only way that PPP gap can close is if the USD appreciates.

To summarize, there are two forces at work. First, the inflation differential implies depreciation of the USD of 2%, whereas the change in the real exchange rate implies appreciation of 3.46%. The latter effect dominates so that overall there is an appreciation, which brings PPP closer to 1.

Problem 3

Suppose Turkey's money growth rate is currently 12% and Turkey's output growth is 5%. Europe's money growth rate is 4% and its output growth is 2%. Also assume Turkey's inflation rate is currently 7% and the world real interest rate is 2%. In what follows, use the conditions associated with the simple monetary model. Treat Turkey as the home country and define the exchange rate as Turkish lira per euro, $E_{\text{TRY / EUR}}$. Assume all trends continue unless stated otherwise.

(a) Compute the nominal interest rate in Turkey.

Solution. The Fisher equation with r^* gives $i_{\text{TRY}} = 2\% + 7\% = 9\%$.

(b) Compute the rate of depreciation in the Turkish lira relative to the euro.

Solution. In this question, we're assuming that all growth rates are the same until stated otherwise. That means there isn't much need to emphasize the importance of expectations here (e.g. Turkish money growth is 12% now and going forward until something changes), so I'll omit the typical expectations superscript.

Anyway, assuming absolute PPP holds, the nominal exchange rate is

$$E_{\text{TRY / EUR}} = \frac{P_{\text{TRY}}}{P_{\text{EUR}}} \quad \Longrightarrow \quad \%\Delta E_{\text{TRY / EUR}} \approx \pi_{\text{TRY}} - \pi_{\text{EUR}} = 7\% - \pi_{\text{EUR}}.$$

But uh, we're not told what euro inflation is. Recall that inflation happens when there's more money chasing fewer goods, so we can calculate euro inflation to be $\pi_{\text{EUR}} = 4\% - 2\% = 2\%$. Okay, problem solved: $\%\Delta E_{\text{TRY / EUR}} = 7\% - 2\% = 5\%$.

The Turkisha lira is expected to depreciate by 5% relative to the euro each year

(c) Suppose the Central Bank of the Republic of Turkey decreases the money growth rate from 12% to 8%. If nothing in Europe changes, what is the new inflation rate in Turkey? What is the new nominal interest rate in Turkey?

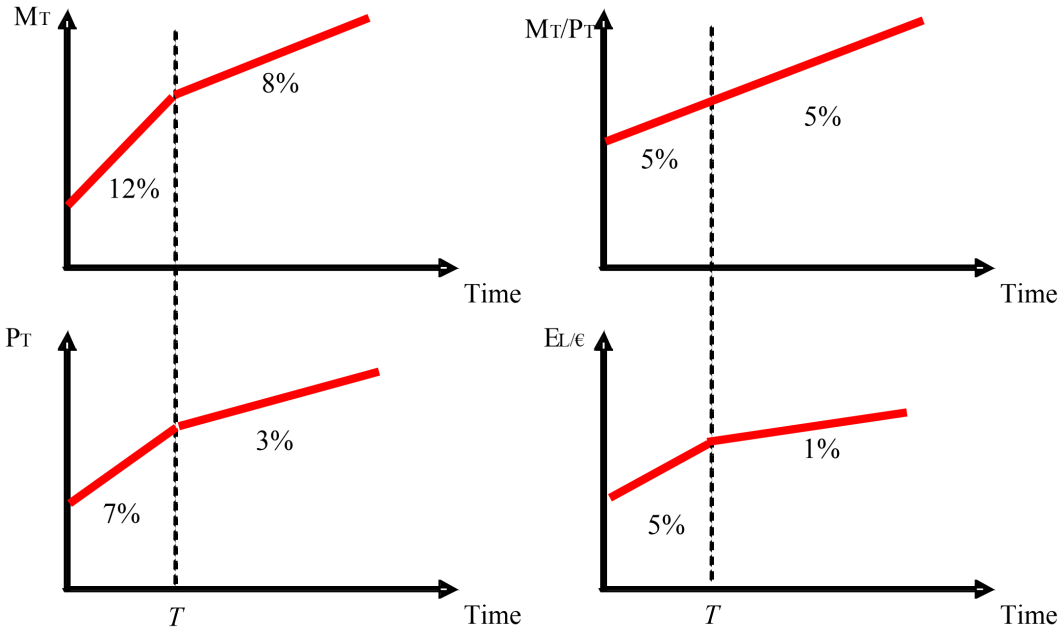
Solution. For the inflation rate, just use the "more money chasing fewer goods" equation and you get $\pi_{\text{TRY}} = 8\% - 5\% = 3\%$. And then use the Fisher equation with the world real interest rate to get the new nominal rate of $i_{\text{TRY}} = 2\% + 3\% = 5\%$.

(d) Using four time series diagrams, illustrate how the change in part (c) affects the following variables over time in Turkey: M_T, real money supply, P_T, and $E_{TRY/EUR}$. The change in the money growth rate occurs at time T.

Solution. The following changes occur:

- The money supply growth rate used to be 12%. Now it's 8%.
- Inflation used to be 7%. Now it's 3%.
- The depreciation rate used to be 5%. Now it's $\%\Delta E_{\text{TRY / EUR}} = 3\% - 2\% = 1\%$.

- The real money supply is defined as M_t / P_t. It follows that the approximate rate of growth in the real money supply is $\mu_t - \pi_t$. The real money supply used to be growing at rate $12\% - 7\% = 5\%$. Now it's growing at rate $8\% - 3\% = 5\%$. No change. This theory relies on PPP, which is a flexible price (or long-run) theory, and money is neutral with perfectly flexible prices (or in the long run). Ergo the change in nominal money growth has no effect on real money growth.



Problem 4

Suppose Turkey's money growth rate is 2% and its output growth is 0%. Europe's money growth rate is 1% and its output growth is 0%. The world real interest rate is 3%. Use the conditions associated with the general monetary model. Treat Turkey as the home country, and define the exchange rate as Turkish lira per euro, $E_{\text{TRY / EUR}}$. Assume all trends continue unless stated otherwise.

(a) Find the rate of inflation, real money balance growth rate, the rate of depreciation, and the nominal interest rate.

Solution. Same equations as before, nothing new. Just note that euro inflation is $\pi_{\text{EUR}} = 1 - 0 = 1\%$. Variables without a subscript are home, i.e. Turkish, variables.

$$\begin{aligned}
\pi = \mu - g &= 2\% - 0\% = 2\%, \\
\text{real money balances growth rate} = \mu - \pi &= 2\% - 2\% = 0\%, \\
\text{depreciation rate} = \pi_{\text{TRY}} - \pi_{\text{EUR}} &= 2\% - 1\% = 1\%, \\
i = r^* + \pi &= 3\% + 2\% = 5\%.
\end{aligned}$$

(b) Suppose at time T, Turkey increases its money growth rate to 3%. Find the rate of inflation, real money balance growth, depreciation, and nominal interest.

Solution. Calculations here are pretty similar, just updated to account for the fact that μ has increased by 1%.

$$\begin{aligned}
\pi = \mu - g &= 3\% - 0\% = 3\%, \\
\text{real money balances growth rate} = \mu - \pi &= 3\% - 3\% = 0\%, \\
\text{depreciation rate} = \pi_{\text{TRY}} - \pi_{\text{EUR}} &= 3\% - 1\% = 2\%, \\
i = r^* + \pi &= 3\% + 3\% = 6\%.
\end{aligned}$$

Note that real money balances are again unaffected via money neutrality.

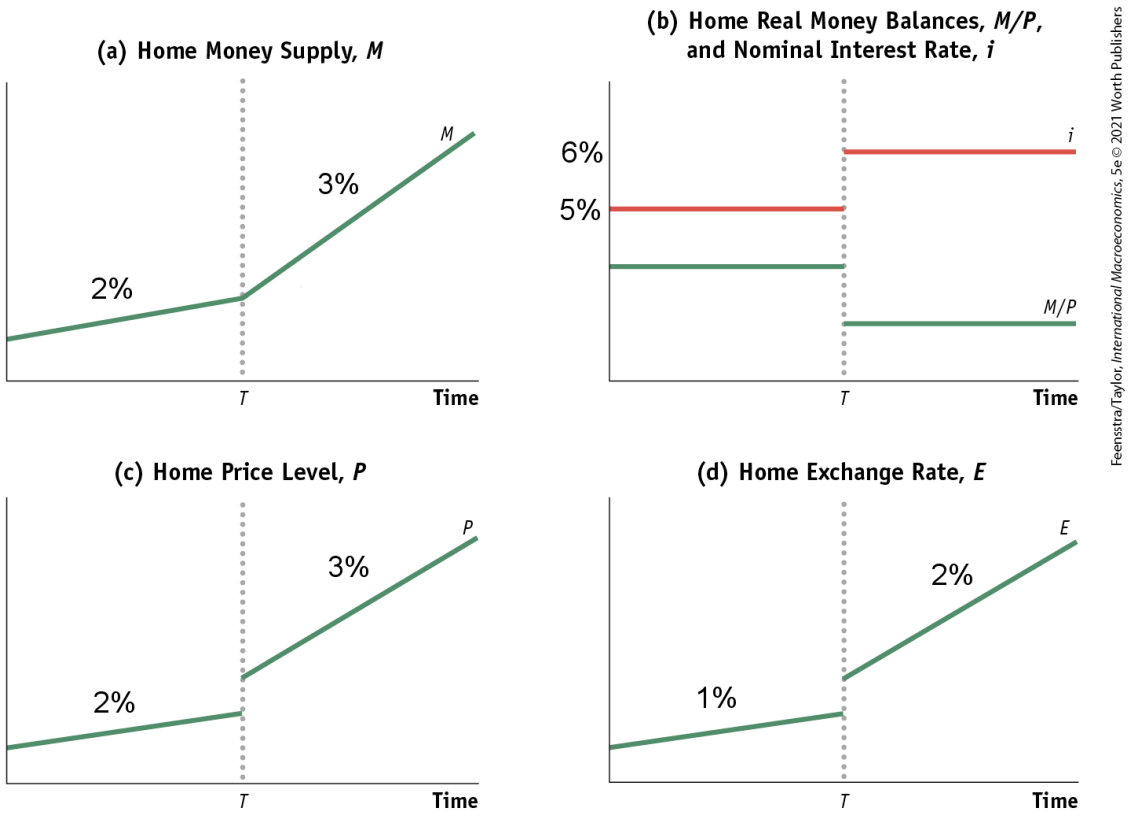
(c) Draw the time series for Turkish M, P, real money balances, E, and i.

Solution. Seems pretty samey to the last question so far, but there's a quirk. Because i increases, $L(i)$ decreases. Which is to say, people want to hold fewer money balances because the opportunity cost of holding money has increased. That means we need M/P to fall.

But notice that $\mu = \pi$, that is, M and P are growing at the same rate. So how can the fraction M/P change if numerator and denominator are both changing at the same rate? We need some other change beyond just looking at rates of change.

In particular, we need a discrete jump in P. This doesn't affect rates of change: a discrete jump has nothing to do with rates of change. But when there is a discrete jump in P, real money balances M/P falls, as needed. And when there is a discrete jump in P, there is also a discrete jump in the exchange rate because $E = P/P_{\text{foreign}}$.

All of this can be seen in the time series plots on the next page. The slopes are all what you'd expect based on the 1% increase in money growth. The novel aspect is the upward jump in i causing the downward jump in M/P, which requires the upward jump in P, which in turn implies the upward jump in E.



Problem 5 (Extra)

Suppose that Vietnam and Côte d'Ivoire produce coffee. The currency unit used in Vietnam is the dong (VND). Côte d'Ivoire is a member of Communauté Financière Africaine (CFA), a currency union of West African countries that use the CFA franc (XOF). In Vietnam, coffee sells for 4,500 VND per pound. The exchange rate is 30 VND per 1 CFA franc, $E_{\text{VND / XOF}} = 30$.

(a) If the LOOP holds, what is the price of coffee in Côte d'Ivoire, measured in XOF?

Solution. The LOOP says that the price of coffee in Côte d'Ivoire should be exactly 4,500 VND after currency conversion if the following conditions also hold:

- absence of trade frictions (such as transport costs and tariffs);
- and under conditions of free competition and price flexibility;
- goods are identical.

If these conditions hold and LOOP holds, then

$$P_C^{\text{coffee}} = 4,500 \cancel{\text{VND}} \times \frac{1\,\text{XOF}}{30\,\cancel{\text{VND}}} = 150\,\text{XOF}\,.$$

(b) Suppose the price of coffee in Côte d'Ivoire is actually 160 CFA francs per pound of coffee. Compute the relative price of coffee in Côte d'Ivoire versus Vietnam.

Where will coffee traders buy coffee? Where will they sell coffee? How will these transactions affect the price of coffee in Vietnam? In Côte d'Ivoire?

Solution. Let Vietnam be the "home" country. The relative price tells us how many pounds of Vietnamese coffee are needed to purchase one unit of Côte d'Ivoire...ian coffee. To make this comparison, we first have to convert everything into the same units/currency, and we may as well use the "home" currency of Vietnam (although you could use the foreign currency as well).

The foreign price goes in the numerator, the home price in the denominator. The foreign price, expressed in VND, is

$$P_C^{\text{coffee}} = 160 \cancel{\text{XOF}} \times \frac{30\,\text{VND}}{1\,\cancel{\text{XOF}}} = 4,800\,\text{VND}\,.$$

Therefore the relative price is

$$q^{\text{coffee}}_{\text{VND / XOF}} = \frac{4,800\,\text{VND}}{4,500\,\text{VND}} = 1.07.$$

The interpretation is that 1.07 pounds of Vietnamese coffee are needed to purchase one pound of Côte d'Ivoirian coffee. Maybe think about it like this. If Vietnam makes one pound of coffee, then it can sell that coffee for 4,500 VND... but that 4,500 VND isn't even enough to buy one pound of Côte d'Ivoirian coffee, even though we're talking about one pound of identical coffee.

That's kinda weird, and it probably won't be like that for long. That's because traders will buy coffee in Vietnam and sell it in Côte d'Ivoire for arbitrage profit. This will cause the price of coffee in Vietnam to increase (because that's where traders are buying tons of it) and the price of coffee in Côte d'Ivoire to decrease (because that's where those traders are selling tons of it). Arbitrage will end when the prices equalize, i.e. when the relative price equals 1.