Problem 1

Consider the country of Nilfgaard, whose currency is the *floren*.

- This year it invests 400 floren in domestic investment, and its previous investments generated 15 floren in capital gains this year.
- It purchases 160 floren worth of foreign assets, and sells 120 floren worth of domestic assets to foreigners.
- Valuation effects lead to a total 5 floren in capital gains.

Supposing KA $= 0$, answer the following things.

(a) What is the change in external wealth?

Solution. External wealth W consists of external assets, A (what the rest of the world owes to the home country) minus external liabilities, L (what the home country owes to the rest of the world). That is, $W = A - L$. There are two things that can affect A and L: the quantity of external assets/liabilities (financial flows) and the value of those external assets/liabilities (valuation effects).

First, financial flows. In this scenario, having acquired foreign assets increases external wealth by 160: Nilfgaard now owns more, say, stock in foreign businesses, so the value of that stock has flowed from foreign to home. Having sold some domestic assets decreases external wealth by 120: Nilfgaard now owns less stock in its own businesses, and the value of that stock has flowed from home to foreign. The net flow affects external wealth to the amount of $160 - 120 = 40$.

The preceding should sound a lot like financial account transactions. Recall that $\text{FA} = \text{EX}_A - \text{IM}_A$. When foreign assets are purchased, home is importing an asset, which is a minus in FA but a plus in W; when domestic assets are sold, home is exporting as asset, which is a plus in FA but a minus in W. Changes in external wealth move in the opposite direction of the financial account, all else equal. Because 120 floren worth of assets have been exported; and 160 floren worth of assets have been imported; we conclude that $\text{FA} = 120 - 160 = -40$.

Now onto valuation effects, which come in through two channels: changes in prices and changes in exchange rates. There was a question in week 2 section where a US citizen had a house in Mexico worth 150,000 MXN. Suppose the exchange rate is and stays at 1 USD per 3.10 MXN, but the price of the house changed to 160,000 MXN. Then the value of the house in USD changes

$$\text{from} \quad 150,000\,\text{MXN} \times \frac{1\,\text{USD}}{3.10\,\text{MXN}} = 48,387\,\text{USD},$$

$$\text{to} \quad 160,000\,\text{MXN} \times \frac{1\,\text{USD}}{3.10\,\text{MXN}} = 51,613\,\text{USD}.$$

This is a capital gain on a home-owned foreign asset ($A \uparrow$) – a positive valuation effect through price effects – of 3,226 USD.

If the price of the house remained at 150,000 MXN but the USD appreciated from 1 USD per 3.10 MXN to 1 USD per 5.64 MXN, then the value of that home-owned foreign asset in USD changes

$$\text{from} \quad 150,000\,\text{MXN} \times \frac{1\,\text{USD}}{3.10\,\text{MXN}} = 48,387\,\text{USD},$$

$$\text{to} \quad 150,000\,\text{MXN} \times \frac{1\,\text{USD}}{5.64\,\text{MXN}} = 26,596\,\text{USD}\,.$$

This is a capital loss on a home-owned foreign asset ($A \downarrow$) – a negative valuation effect through exchange rate effects – of 21,791 USD. Extend the same logic to foreign-owned home assets (i.e. changes in L).

The 5 floren in valuation effects means that, overall, the foreign assets owned by home have gained more value than the home assets owned by foreigners. So the total change in external wealth is

$$\begin{aligned} \Delta W &= -\text{FA} + \text{valuation effects} \\ &= 40 + 5 \\ &= 45 \text{ floren.} \end{aligned}$$

(b) What is the current account?

Solution. We know that $\text{CA} + \text{FA} + \text{KA} = 0$, and it follows that $-\text{FA} = \text{CA} + \text{KA}$. Since $\text{KA} = 0$, it follows that $\text{CA} = 40$ floren. This is a current account surplus: expenditure on goods and services is less than what their income resources could allow. Letting VE denote valuation effects, we can write more generally

$$\Delta W = \text{CA} + \text{KA} + \text{VE}.$$

(c) What is the total change in wealth?

Solution. Total wealth is domestic wealth plus external wealth, big surprise. Domestic wealth increases by 400 floren from investing in new home capital (like a new home factory); and by the 15 floren in home capital gains (which is a domestic valuation effect). So the total change in wealth is $45 + (400 + 15) = 460$ floren.

(d) What is the amount of domestic savings?

Solution. In an open economy, $S - I = \text{CA}$. We can again interpret this as saying that the CA represents *unspent income*: if CA is positive, then $S > I$, so more is being saved. Anyway,

$$S = \text{CA} + I = 40 + 400 = 440 \text{ floren.}$$

Problem 2 (Sample Midterm 2, Question 3)

Suppose that in a typical year, a country produces $Q = 50$ output with $C = 50$ and $I = G = 0$. The country has no initial wealth, and the world interest rate is $r^* = 10\%$.

(a) There is an unexpected drop in output in year $t = 0$, so output falls to 39 and is then expected to return to 50 in every future year. If the country desires to smooth consumption, how much should it borrow in period $t = 0$? What will the new level of consumption be from then on?

Solution. With zero initial wealth, the long-run budget constraint (LRBC) is

$$\text{PV}(Q) = \text{PV}(C) + \text{PV}(I) + \text{PV}(G),$$

which says that expenditure, when all is said and done, must equal production. The idea is that flows-in of resources have to equal flows-out of resources over time.

Output is hit with a negative shock and drops to 39 in period 0, and then rebounds to 50 for every subsequent period forever. Because the world interest rate is 10%, it follows that the present value of output is

$$\begin{aligned}\text{PV}(Q) &= 39 + \left[\frac{50}{1.10} + \frac{50}{1.10^2} + \frac{50}{1.10^3} + ...\right] \\ &= 39 + \left[\frac{50}{0.10}\right] \\ &= 539.\end{aligned}$$

Refer to week 1 problem 4d for a refresher on calculating present values if needed.

Since we're told $I = G = 0$, the LRBC only requires $\text{PV}(C) = \text{PV}(Q) = 539$. In words: there's 539 worth of resources (in present value terms) that can be divvied up and devoted to consumption in some way over time. For example, they could consume 539 in period $t = 0$ (by borrowing a ton), and then never consume anything ever again because all of their future resources must be devoted to paying back that initial huge loan. Seems extreme. Or they could consume 39 in period $t = 0$ and 50 in every subsequent period.

Or they could divide up that 539 equally over each period: they could *smooth* consumption. Indeed, people typically would rather reduce consumption by a little bit indefinitely than take a really huge hit to consumption this year.

Which is to say, we need to find some constant value of C such that taking the present value of that C over all periods equals 539. In maths, we need to solve

$$C + \left[\frac{C}{1.10} + \frac{C}{1.10^2} + \frac{C}{1.10^3} + ...\right] = C + \left[\frac{C}{0.10}\right] = 539,$$

which can be solved for $C = 49$.

In response to the period $t = 0$ drop of output by 11, the country borrows in period 0 so that it's consumption only falls to 49 (instead of falling to 39). The loan has to be paid back in future periods, so consumption is indefinitely reduced by 1 each year.

There's a little formula that can be used to calculate the change in consumption,

$$\Delta C = \left[\frac{r^*}{1+r^*}\right]\Delta Q \quad \Longrightarrow \quad \Delta C = \left[\frac{0.10}{1.10}\right](-11) = -1.$$

I won't derive this formula (it's in the book), but notice that the fraction $r^*/(1+r^*)$ is increasing in r^*. Mathematically, when r^* is large, ΔC will be closer to ΔQ. The interpretation is intuitive: there will be a bigger change in C when it costs a lot to borrow, because consumption smoothing (through borrowing) becomes relatively expensive via large future interest payments.

Let's conclude. In period $t = 0$, output will be 39 and consumption will be 49. That extra 10 consumption must have been imported, so TB $= -10$. Funding that extra 10 consumption comes from borrowing in period $t = 0$, requiring FA $= 10$ as the country exports e.g. home bonds to fund those imports. It follows that CA $= -10$ entirely through the trade balance.

In all subsequent periods, consumption is now always 1 lower than its previous level of 50. Think about it: 10 is borrowed in period $t = 0$, and the interest rate is 10%, so every subsequent period the country has to pay back $(0.10)10 = 1$. Ergo the indefinite reduction of future consumption by 1 every year, which becomes income of the foreign country, technically NFIA $= -1$. And in all future years, 50 is produced but only 49 is consumed, so that must mean the remaining 1 is exported to another country (which is how the interest payment is funded): TB $= 1$, and therefore CA $= 0$ with nothing going on in the financial account.

Finally, notice that the present value of the trade balance is

$$\text{PV(TB)} = -10 + \left[\frac{1}{0.10}\right] = 0.$$

Short-run deviations from exporting and importing balance out to zero eventually.

t	0	1	2	3	...	PV
Q	39	50	50	50	...	539
C	49	49	49	49	...	539
TB	−10	1	1	1	...	0
NFIA	0	−1	−1	−1	...	—
CA	−10	0	0	0	...	—
FA	10	0	0	0	...	—

(b) There is an unexpected war in year $t = 0$, which costs 11 units and is predicted to last one year. If the country desires to smooth consumption, how much should it borrow in year $t = 0$? What will the new level of consumption be from then on?

Solution. There is no shock to output, so the present value of output is

$$\text{PV}(Q) = 50 + \left[\frac{50}{0.10}\right] = 550.$$

There is no investment expenditure, so the long-run budget constraint only requires

$$\text{PV}(C) + \text{PV}(G) = \text{PV}(Q) = 550.$$

The war is (thought to be) a one-time government expenditure in period $t = 0$, so $\text{PV}(G) = 11$. Therefore we must have

$$\text{PV}(C) + 11 = 550 \quad \Longrightarrow \quad \text{PV}(C) = 539.$$

In words: there's 539 worth of resources (in present value terms) that can be divvied up and devoted to consumption in some way over time. This sounds familiar.

Consumption smoothing satisfies

$$C + \left[\frac{C}{0.10}\right] = 539 \quad \Longrightarrow \quad C = 49.$$

Same thing in the previous part: there was a one-time shock that reduced period $t = 0$ resources available for consumption by 11 units; but in order to smooth consumption, people would rather borrow now and slightly reduce future consumption indefinitely. There is analogous formula,

$$\Delta C = -\left[\frac{r^*}{1 + r^*}\right] \Delta G \quad \Longrightarrow \quad \Delta C = -\left[\frac{0.10}{1.10}\right](11) = -1.$$

Let's conclude. In period $t = 0$, output will be 50, consumption will be 49, government spending will be 11, so total expenditure will be 60. That extra 10 consumption must have been imported, so $\text{TB} = -10$. Funding that extra 10 consumption comes from borrowing in period $t = 0$, requiring $\text{FA} = 10$ as the country exports e.g. home bonds to fund those imports. It follows that $\text{CA} = -10$ entirely through the trade balance.

In all subsequent periods, consumption is now always 1 lower than its previous level of 50. This is because 10 is borrowed in period $t = 0$, and the interest rate is 10%, so every subsequent period the country has to pay back $(0.10)10 = 1$. Ergo the indefinite reduction of future consumption by 1 every year, which becomes income of the foreign country, technically $\text{NFIA} = -1$. And in all future years, 50 is produced but only 49 is consumed, so that must mean the remaining 1 is exported to another country (which is how the interest payment is funded): TB $= 1$, and therefore CA $= 0$ with nothing going on in the financial account.

Finally, notice that the present value of the trade balance is

$$\text{PV}(\text{TB}) = -10 + \left[\frac{1}{0.10}\right] = 0.$$

In the long-run, when all is said and done, the flows in of resources have to equal the flows out. The déjà vu is making my brain melt.

t	0	1	2	3	...	PV
Q	50	50	50	50	...	550
C	49	49	49	49	...	539
G	11	0	0	0	...	11
TB	-10	1	1	1	...	0
NFIA	0	-1	-1	-1	...	—
CA	-10	0	0	0	...	—
FA	10	0	0	0	...	—

(c) Following part (b), the country wakes up in year 1 and discovers that the war is still going on and will eat up another 11 units of expenditure in year 1. If the country still desires to smooth consumption looking forward from year 1, how much should it borrow in period 1? What will the new level of consumption be from then on?

Solution. There's a long way to do this, crossing every x and dotting every j. But there's a simpler, more conceptual way of thinking about it. I'll do the latter.

When the government spent $G = 11$ in period $t = 0$, it ultimately caused C to fall by 1 (from 50 to 49) in that and all subsequent years because the country borrowed 10 and therefore had to make an interest payment of 1 in all future periods.

Now just repeat the same logic, starting in period $t = 1$. When the government spends $G = 11$ in period $t = 1$, it ultimately causes C to go down by *another* 1 (from 49 to 48) in that and all subsequent years because the country borrows another 10 and therefore has to make an *additional* interest payment of 1 in all future periods.