Problem 1 (Sample Midterm 2, Question 2)

It is year $t = 0$. Argentina thinks it can find \$150 of domestic investment projects with an MPK of 10%. Argentina invests \$84 in year $t = 0$ by borrowing \$84 from the rest of the world at the world interest rate $r^* = 5\%$. There is no further borrowing or investment. The project starts to pay off in year $t = 1$ and continues to pay off all years thereafter. Interest is paid in perpetuity, in year $t = 1$ and every year thereafter. In addition, assume that if the projects are not done, then GDP $= Q = C =$ \$200 in all years.

For the following questions, use standard assumptions: initial external wealth $W = 0$, $G = 0$ always, $I = 0$ except in year $t = 0$, and NUT $=$ KA $= 0$; and furthermore there is no net labor income so that NFIA $= r^*W$.

(a) If the investment project is not undertaken, what is the present value of output Q?

Solution. \$200 is earned every year, and all subsequent years must be discounted by the interest rate $r^* = 5\%$. Using the present value formula (see week 1 section problem 4d if you need a refresher), we get

$$\begin{aligned} \text{PV}(Q) &= 200 + \left[\frac{200}{1.05} + \frac{200}{1.05^2} + \frac{200}{1.05^3} + ...\right] \\ &= 200 + \left[\frac{200}{0.05}\right] \\ &= 4200. \end{aligned}$$

(b) Should Argentina fund the \$84 worth of projects? Explain your answer.

Solution. A project is worth funding if the MPK exceeds the interest rate, that is, if the payoff of the investment exceeds the cost of the investment. Here we have

$$\text{MPK} = 10\% > 5\% = r^*,$$

so yeah, it should invest.

(c) Why might Argentina be able to borrow only \$84 and not \$150?

Solution. Some countries face borrowing limits, especially those with sketchy financial situations or histories. Argentina for example is the modern poster child for economic dysfunction: it has defaulted on its debts *nine times* since independence from Spain in 1816, and is under threat of another default as I write this. Would *you* want to loan to Argentina?

(d) Going forward, assume the projects totaling \$84 are funded and completed in year $t = 0$. If the MPK is 10%, what is the total payoff from the projects in future years?

Solution. Output is initially at $Q = 200$. An MPK of 10% means that an increase in K of 1 unit will lead to an increase in Q of 0.10 units. We're told that the increase in K is 84, therefore the increase in output is 8.4 in each subsequent year.

(e) At year $t = 0$, what is the new PV(Q), PV(I), and PV(C)?

Solution. GDP will be 200 in year $t = 0$, before the investment project is completed. Then in all subsequent years, GDP will be 208.4. Therefore the present value calculation gives

$$\begin{aligned}\text{PV}(Q) &= 200 + \left[\frac{208.4}{1.05} + \frac{208.4}{1.05^2} + \frac{208.4}{1.05^3} + \dots\right] \\ &= 200 + \left[\frac{208.4}{0.05}\right] \\ &= 4368.\end{aligned}$$

In year $t = 0$, the investment of \$84 is undertaken. Then no more investment ever. So PV$(I) = 84$. Under the long-run budget constraint, the present value of consumption and investment must equal the present value of output, that is,

$$\text{PV}(C) + \text{PV}(I) = \text{PV}(Q) \quad \Longrightarrow \quad \text{PV}(C) + 84 = 4368 \quad \Longrightarrow \quad \text{PV}(C) = 4284.$$

(f) Suppose Argentina is consumption smoothing. What is the percent change in PV(C)? What is the new level of C in all years? Is Argentina better off?

Solution. We want to find some constant stream of consumption C that has present value PV$(C) = 4284$. We can write such a stream as

$$\begin{aligned}\text{PV}(C) &= C + \left[\frac{C}{1.05} + \frac{C}{1.05^2} + \frac{C}{1.05^3} + \dots\right] \\ &= C + \left[\frac{C}{0.05}\right] \\ &= \left(\frac{1.05}{0.05}\right) C.\end{aligned}$$

So we want to solve

$$\left(\frac{1.05}{0.05}\right) C = 4284 \quad \Longrightarrow \quad C = 204.$$

Absent investment, it would have only $C = Q = 200$ in every period.

(g) In year $t = 0$, when the investment project is started (but not yet completed), explain Argentina's balance of payments as follows: state CA, TB, NFIA, and FA.

Solution. In year $t = 0$, output is $Q = 200$, consumption is $C = 204$, and investment is $I = 84$. Clearly $C + I > Q$, i.e. expenditure exceeds output by 288 versus 200, so Argentina must be borrowing FA $= 88$ by e.g. exporting bonds. And therefore

it must also be a current account deficit, CA $= -88$, because they're using more resources than they've produced.

They don't have to pay anything back until subsequent years, so NFIA $= 0$. This implies that TB $= -88$ since NUT $=$ NFIA $= 0$ implies CA $=$ TB.

(h) State the levels of CA, TB, NFIA, and FA in year $t = 1$ and every later year.

Solution. In subsequent years, output is $Q = 208.4$, consumption is 204, and no more investments are being made so $I = 0$. Now we have $C + I < Q$, i.e. expenditure falls short of output. No borrowing or lending is occurring anymore, so FA $= 0$. But the original loan now requires interest payments.

The loan was for 88 and the interest rate is 5%, so Argentina pays back $(0.05)88 = 4.4$ in interest every year, that is, NFIA $= -4.4$ each year. Also TB $= Q - C - I = 4.4$. Intuitively, Argentina is consuming less than its resources and exporting the extra to pay back the loan it took in period 0.

t	0	$1, 2, 3, \ldots$	PV
Q	200	208.4	4368
I	84	0	84
C	204	204	4284
TB	-88	4.4	0
NFIA	0	-4.4	—
CA	-88	0	—
FA	88	0	—