Problem 1 (Sample Final Question 5)

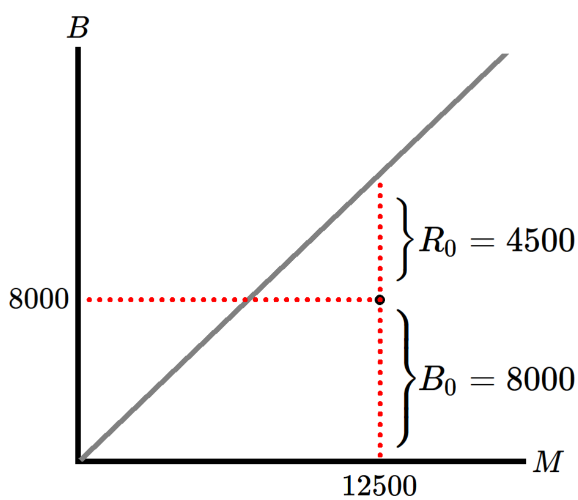
The Republic of Banania is currently pegging the Bananian peso to the dollar at $E = 1$ peso per dollar. In year 0 the money supply is $M = 12500$ pesos, reserves are $R = 4500$ pesos, and domestic credit is $B = 8000$ pesos. To finance spending, B is growing at 25 percent per year. Inflation is currently zero, prices are flexible, PPP holds at all times, and initially $P = 1$. Assume also that the foreign price level is $P^* = 1$, so PPP holds. The government will float the peso if and only if it runs out of reserves. The US nominal interest rate is $i^* = 5\%$. Real output is fixed at $Y = 12500$ at all times. Real money balances are initially $M/P = 12500 = L(i)Y$, when $i = 5\%$, and L is initially equal to 1.

(a) Assume that Bananian investors are myopic and do not foresee the reserves running out. Compute domestic credit in years 0, 1, 2, 3, and 4. At each date, also compute reserves, money supply, and the growth rate of money supply since the previous period (in percent).

Solution. First, let's put a bit of a story to this. The government wants to spend a certain amount, but its own funding falls short by 8000. To make up for that 8000 shortfall, the treasury will sell $B = 8000$ worth of bonds to the central bank in exchange for cash. (The central bank can afford to purchase these 8000 worth of bonds by essentially just creating cash with a few mouse clicks. Yes, that *should* make you uncomfortable, but not necessarily terrified.[1]) The point is, 8000 cash is entering circulation, which is part of the money supply.

The central bank also has 4500 worth of foreign currency as reserves. In order to acquire that foreign currency, the central bank exchanged its own currency. The point is, $R = 4500$ cash is entering circulation, which is part of the money supply.

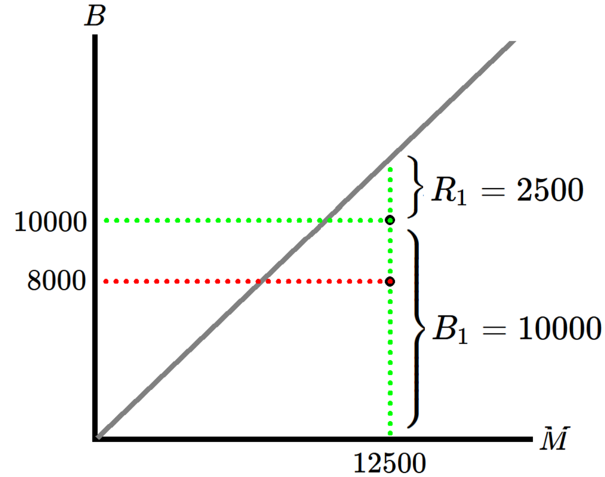
Let's make a table and central bank balance sheet diagram. The x-axis shows the money supply, and y-axis the supply of domestic credit. Because $M = B + R$, it follows that $B = M$ (i.e. the 45 degree line) when $R = 0$, that is, when reserves have run out. We're are told the numbers for Year 0, so just plug things in.



Year	B	R	M	μ
0	8000	4500	12500	–

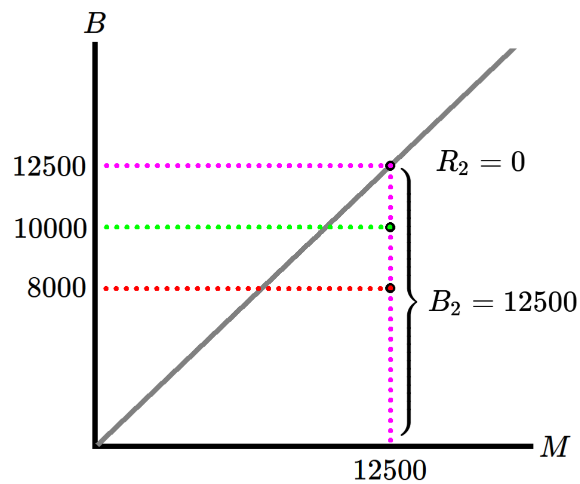
[1] https://www.usatoday.com/in-depth/money/2020/05/12/coronavirushow-u-s-printing-dollars-save-economy-during-crisis-fed/3038117001/

- Year 1: B grows at rate 25%, so $B = 8000(1.25) = 10000$. To maintain the peg, the central bank must hold M constant, and they do this by reducing R by 2000.



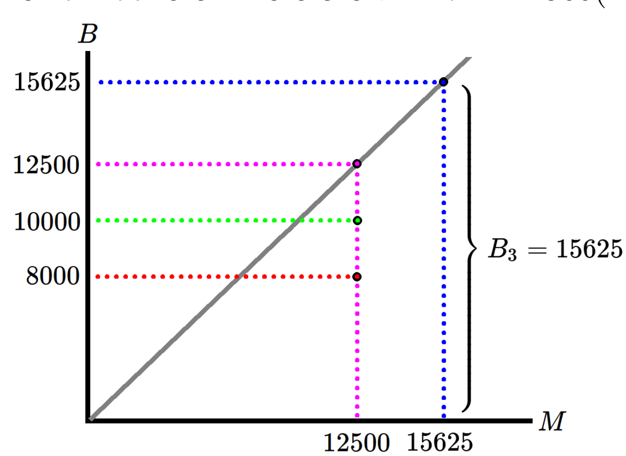
Year	B	R	M	μ
0	8000	4500	12500	–
1	10000	2500	12500	0

- Year 2: B grows to $10000(1.25) = 12500$. This means R must shrink by 2500 to maintain constant M. Uh oh, reserves are zero: the central bank can no longer run down its reserves in order to maintain a constant money supply.



Year	B	R	M	μ
0	8000	4500	12500	–
1	10000	2500	12500	0
2	12500	0	12500	0

- Year 3: Okay, now when B increases by 25%, so must M, so $\mu = 25\%$. Reserves remain at zero. Therefore $B = M = 12500(1.25) = 15625$.



Year	B	R	M	μ
0	8000	4500	12500	–
1	10000	2500	12500	0
2	12500	0	12500	0
3	15625	0	15625	25%

- Year 4: And repeat. B and M increase to $15625(1.25) = 19531.25$, so $\mu = 25\%$. Reserves remain at zero.

Year	B	R	M	μ
0	8000	4500	12500	–
1	10000	2500	12500	0
2	12500	0	12500	0
3	15625	0	15625	25%
4	19531.25	0	19531.25	25%

(b) Continue to assume myopia. When do reserves run out? Call this time T. Assume inflation is constant after time T. What will that new inflation rate be? What will the rate of depreciation be? What will the new domestic interest rate be?

Solution. As seen above, reserves run out in year $T = 2$. Then the nominal money supply instantaneously begins growing at 25%, so inflation instantaneously jumps to 25% and the rate of depreciation instantaneously jumps to 25% too. Inflation was initially zero when $i = 5\%$, implying $r = 5\%$. But now we have non-zero inflation, so the nominal interest rate instantaneously jumps to

$$\begin{aligned} i &= r + \pi \\ &= 5\% + 25\% \\ &= 30\%. \end{aligned}$$

The change in i will have implications in the money market, shown next.

(c) Continue to assume myopia. Suppose that at time T, when the home interest rate i jumps up, $L(i)$ drops from 1 to $4/5$. Recall that Y remains fixed. What is M/P before time T? What will be the new level of M/P after time T, once reserves have run out and inflation has started?

Solution. Before time T, we have $M = 12500$ and $P = 1$ so $M/P = 12500$.

Then because $R = 0$ at time T, we must have $M = B = PL(i)Y$. Plugging in what we know and are told about $T = 2$, we have

$$\frac{M}{P} = \frac{12500}{P} = \frac{4}{5} \times 12500 = 10000.$$

Alright, so the new equilibrium $i = 30\%$ as determined by the Fisher equation tells us that equilibrium real money demand – and therefore supply – instantaneously changes to 10000.

(d) Continue to assume myopia. At time T, what is the price level going to be right before reserves run out? Right after? What is the percentage increase in the price level? In the exchange rate?

Solution. Before time T, the price level stays at $P = 1$ because the nominal money supply is unchanged; inflation is zero and the interest rate remains at $i = 5\%$ as given. But immediately as T hits, we know that $M/P = 10000$ and we know that $M = 12500$ from parts b and c, from which it follows that $P = 1.25$. Therefore the increase in the price level is 25%.

The exchange rate under PPP is $E_{H/F} = P_H/P_F$. There has been no change to P_F, but P_H has jumped by 25%, therefore $E_{H/F}$ must also jump by 25% so $E_{H/F} = 1.25$.

Okay, that's a lot to take in. It might help here to draw the money market graph, shown below. I'll repeat the steps in bullet points to hopefully make each logical increment easier to digest.

- At time $T = 2$, $M = 12500$ and $R = 0$, which means $B = M = 12500$.
- Because B is growing at rate 25%, it follows that M is now growing at rate 25%, and therefore inflation instantaneously jumps from 0% to 25%.
- The Fisher equation says $i = r + \pi$. We went from $i = 5\% + 0\%$ to an instantaneously jump of $i = 5\% + 25\% = 30\%$.
- When the instantaneous jump in the interest rate causes $L(i)$ to drop to 4/5, it means we're moving *along* the money demand function (because i is on the vertical axis of the money market graph) from point A (12500) to point B (10000).
- So there must be an instantaneous shift left of the real money supply to get to that new equilibrium $i = 30\%$. Well, we know that M is still 12500, so the only way there can be an instantaneous decrease in M/P is if P instantaneously jumps from 1 to 1.25. (Note that this instantaneous change in P is distinct from inflation: inflation is a *rate of change over time*, whereas here the increase in P is an instantaneous, one-off jump.)

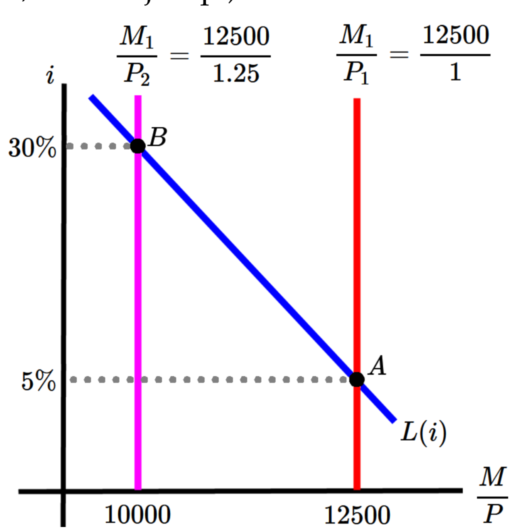


FIGURE 1: There is a sudden increase in P to equilibrate real money supply with real money demand at the new (and sudden) interest rate of 30%.

This is unlikely to happen in practice, however. People who have pesos are likely to anticipate the expected depreciation and therefore will likely want to get rid of pesos in exchange for dollars before that happens. In this case, investors are forward-looking instead of myopic. Which leads to the next question...

(e) Suppose investors know the rate at which domestic credit is growing. Given the above data, when do you think a speculative attack would occur? At what level of reserves will such an attack occur? Explain your answer.

Solution. A major concern with the previous problem is that there was a jump in the price level once reserves hit zero, which in turn caused the jump in the exchange rate. This jump in the price level was necessary because the real money supply M/P needed to fall to 10000 in order to equilibrate at $i = 30\%$, but M was fixed.

But if peso-holders are forward-looking, then they're not just going to sit around and wait for the exchange rate to suddenly jump: they're going to unload some of those pesos before the pesos suddenly and dramatically depreciate. To that end, what if speculators instead make M jump downwards via speculative attack? Then M/P can fall as needed with no jump in P and therefore no jump in E.

To illustrate, let's look at year 1. The central bank has 2500 pesos worth of dollar reserves (which also happens to be 2500 dollars since the exchange rate is assumed to be $E = 1$). If speculators attack now and sell 2500 pesos to the central bank in exchange for 2500 dollars, then the peg can no longer be maintained because $R = 0$.

- Then we have $M = B = 10000$ and $R = 0$.
- Because B is growing at 25%, it follows that is M now growing at 25%, and therefore inflation instantaneously jumps from 0% to 25%.
- The Fisher equation says $i = r + \pi$. We went from $i = 5\% + 0\%$ to an instantaneously jump of $i = 5\% + 25\% = 30\%$.
- When the instantaneous jump in the interest rate causes $L(i)$ to drop to $4/5$, it means we're moving *along* the money demand function (because i is on the vertical axis of the money market graph) from point A (12500) to point B (10000).
- So there must be an instantaneous shift left of the real money supply to get to that new equilibrium $i = 30\%$. But wait, this time $M/P = 10000/1$ is already at the equilibrium level because of the speculative attack, and therefore there is no need for P to change. P doesn't make a jump, so neither does E.

All of this stuff about sudden depreciation and speculative attacks and so forth implies a riskiness to holding foreign currency that we haven't talked much about. Which is to say, we should expect riskier currencies to offer a higher rate of return to accommodate added riskiness. We can write a more general form of UIP as

$$i = i^* + \left(\frac{E^e}{E} - 1\right) + \text{exchange rate RP} + \text{default RP}.$$

In words: the payoff of investing with a foreign currency must account for expected depreciation, uncertainty about your expectation (risk aversion), and the risk of default. For example, you might expect depreciation to be 0 percent, but you fear the *possibility* that depreciation could actually be 20 percent, and want to be compensated for that that fear. Expected depreciation plus the exchange rate RP are called the *currency premium*, and the default risk premium is called the *country premium*.

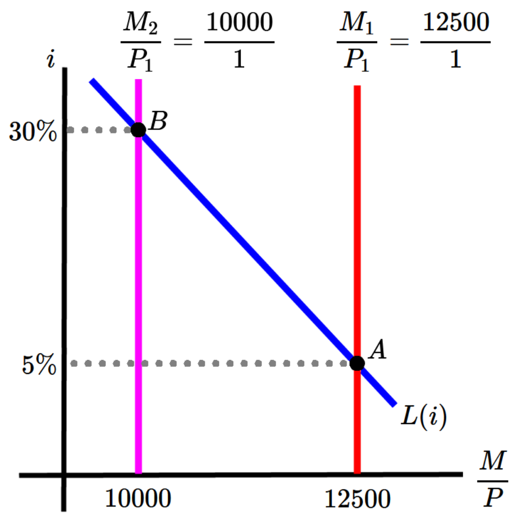


FIGURE 2: When the peg breaks, M/P will fall by 2500 no matter what. Peso holders can avoid a capital loss by draining the central bank's reserves exactly when $R = 2500$, which ensures that P and therefore E do not make any jumps.

Problem 2

Evaluate how the following shocks affects a country's ability to defend a peg.

(a) The central bank sells government bonds.

Solution. When the central bank sells bonds, it is reducing B; to maintain fixed M, it must therefore increase R. When it has more R, it is better able to defend its peg.

(b) Currency traders expect a depreciation in the home currency in the future.

Solution. Holders of home currency will try to sell it before it depreciates. This means the central bank will end up buying home currency in exchange for foreign currency. The central bank's loss of foreign currency holdings is a decrease in reserves, which makes it more difficult to maintain a peg.

(c) The foreign interest rate falls.

Solution. If the foreign interest rate falls, then the FX market implies an appreciation of Home currency (shift the FR curve down). To maintain the peg, the central bank must get back to the initial exchange rate, which in requires a lower interest rate, which the central bank can achieve by increasing the money supply. One way it can increase the money supply is by selling Home currency (which would therefore now be in circulation) for Foreign currency, which it holds as reserves R. More reserves makes it easier to defend the peg.