

# 1 Motivation: Laffer Curve

Let's think about income taxes for a second. Suppose the tax rate is zero. Then the government will not collect any tax revenue because it's not taxing anyone. When it increases the tax rate a little bit, though, it will generate some tax revenue because now it's, you know, actually taxing people.

Now instead suppose the tax rate is 100 percent. Because the government is taking away 100 percent of everyone's income, people are not going to bother working, and no one has any income for the government to tax. So tax revenue is zero.

Point is, the level of income tax revenue generated is plausibly a nonlinear function of the income tax rate that looks like the figure below, which is called the *Laffer curve*. That's the theory, anyway.

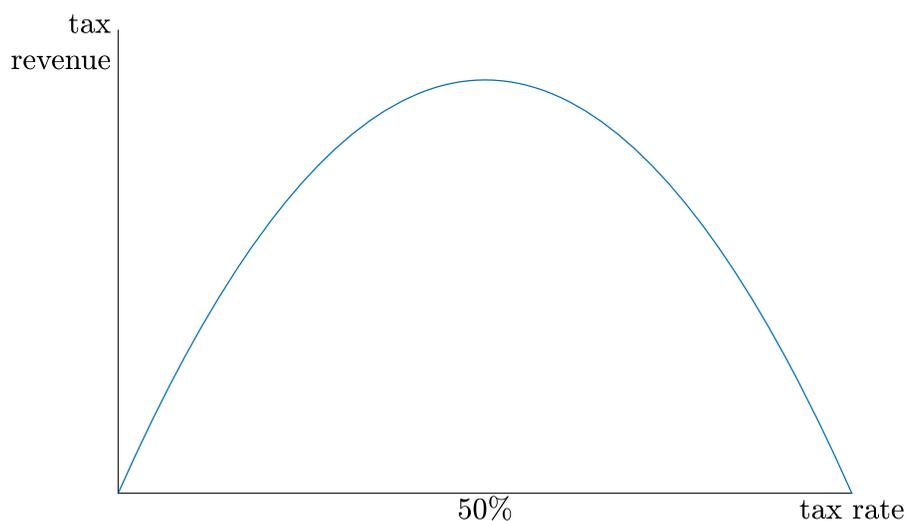


FIGURE 1: When the tax rate is low, more tax revenue can be generated by increasing the tax rate. When the tax rate is too high, however, working will be disincentivized, taxable income will fall, and therefore tax revenue will fall. Tax revenue is maximized at the vertex.

Let  $r$  denote income tax revenue and  $t$  denote the income tax rate. The Laffer curve can be captured via regression of form

$$r = \beta_1 + \beta_2 t + \beta_3 t^2 + \beta_4 z + u, \quad (1)$$

where  $z$  is just some variable (or variables) that we want to control for (don't worry too much about  $z$ ). We know  $\beta_1 = 0$  because we know that a tax rate of zero implies zero tax revenue. (This is one of those cases where it's okay to not estimate the intercept coefficient.) Because the parabola is upside-down, we also expect  $\beta_3 < 0$ .

Now suppose we run the regression and come up with the estimate

$$\hat{r} = 500t - 5t^2 + 9z. \quad (2)$$

The vertex is given by  $t = 50$ , which means that tax revenue is maximized at a tax rate of 50 percent.<sup>1</sup> If the tax rate is below 50%, then increasing the tax rate can generate more tax revenue. If the tax rate is above 50%, then increasing the tax rate will reduce tax revenue.

The marginal effect tells us how much higher we expect tax revenue to be if the tax rate is a little bit higher, with all other regressors held constant (in this case, holding  $z$  constant). In maths, the marginal effect is given by

$$\frac{\partial \hat{r}}{\partial t} = 500 - 10t.$$

This equation makes it clear that if the tax rate is below  $t = 50$  percent, then the marginal effect of tax rate on tax revenue will be positive. And if the tax rate is above  $t = 50$  percent, then the marginal effect of tax rate on tax revenue will be negative.

For example, if a country has a tax rate of  $t = 20$  percent, then their marginal effect is

$$\frac{\partial \hat{r}}{\partial t}(20) = 500 - 10(20) = 300.$$

This says that a country with a tax rate one percentage point higher than 20, other things held equal, on average has tax revenue of about 300 more.

Or if a country has a tax rate of  $t = 70$  percent, then their marginal effect is

$$\frac{\partial \hat{r}}{\partial t}(70) = 500 - 10(70) = -200.$$

This says that a country with a tax rate one percentage point higher than 70, other things held equal, on average has tax revenue of about 200 less.

In general, the **marginal effect** of regressor  $x_j$  on dependent variable  $y$  is given by

$$ME \equiv \frac{\partial \hat{y}}{\partial x_j}. \quad (3)$$

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<sup>1</sup>This value actually seems to be closer to 70 percent in reality, although you should not be lulled into thinking that a 70 percent tax rate is necessarily the socially optimal tax rate. It is of critical importance is how the tax revenue is used. Addressing homelessness in an intelligent way? Great. Building a golden statue of Dear Leader? Not so great.

## 2 Average Marginal Effect (AME)

The marginal effect depends on  $t$ , so it's not possible to capture the entirety of marginal effects of the data in a single number. But we can still use some traditional summary statistics to capture some overall picture of what marginal effects are in the data.

Suppose we have three countries, shown in the figure below. (In practice we'd need a lot more than three countries to establish the upside-down parabola relationship, but doing the calculations by hand for more than three countries would be more painful than useful.)

- Country 1 has a tax rate of 25 percent and collects 8000 tax revenue (the blue dot). The marginal effect for country 1 is

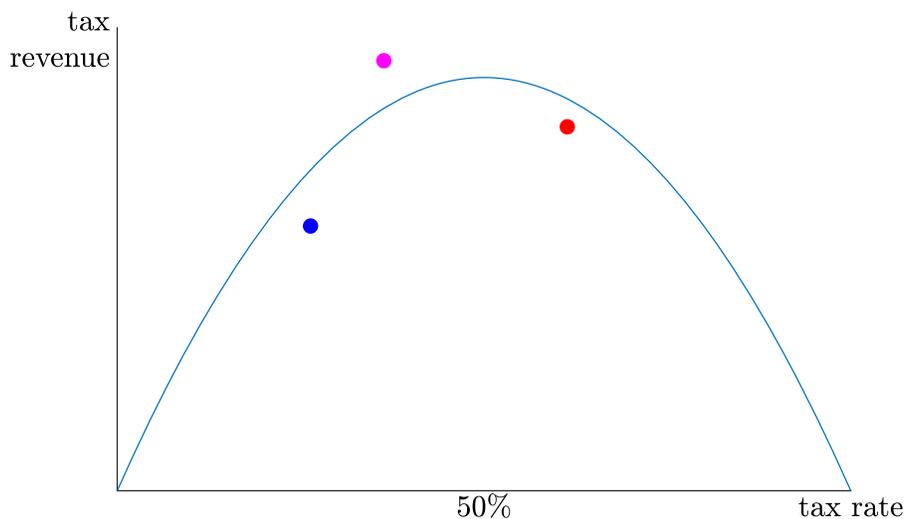
$$ME_1 = 500 - 10(25) = 250.$$

- Country 2 has a tax rate of 35 percent and collects 13000 tax revenue (the magenta dot). The marginal effect for country 2 is

$$ME_2 = 500 - 10(35) = 150.$$

- Country 3 has a tax rate of 60 percent and collects 11000 tax revenue (the red dot). The marginal effect for country 3 is

$$ME_3 = 500 - 10(60) = -100.$$



Okay, so we have three observations and we've calculated the marginal effect for those three observations. I should not have to convince you then that the average marginal effect is given by

$$\text{AME} = \frac{250 + 150 - 100}{3} = 100.$$

It's really just like any other average, except the thing we're averaging is marginal effects, hence its name.

The general definition of the **average marginal effect (AME)** of regressor  $x_j$  on  $y$  (when there are no interaction terms) is

$$\text{AME} \equiv \frac{1}{n} \sum_{i=1}^n \frac{\partial \hat{y}}{\partial x_j}(x_i) \quad (4)$$

Again in words: evaluate the marginal effect at all  $n$  observations, and average them. Geometrically, it is the average slope of the regression for our data, which would be difficult for me to draw and for you to look at so I'm not even going to try.

This gives an overall picture of marginal effects in the data. You can see in the figure that two observations are below 50 percent, and only one observation is above 50 percent, so it shouldn't be too surprising that the average marginal effect is positive.

### 3 Marginal Effect at the Mean (MEM)

Another way to try to get an overall idea of what marginal effects are in the data is to evaluate the marginal effect at the mean of the regressor. Here the mean tax rate is

$$\bar{t} = \frac{25 + 35 + 60}{3} = 40.$$

So the marginal effect evaluated at the mean is

$$\text{MEM} = 500 - 10(40) = 100.$$

Oh hey, it's exactly the same as the average marginal effect. This is the case when the regressor of interest is quadratic, but it is not true in general that  $\text{MEM} = \text{AME}$ . The MEM is illustrated in the figure below.

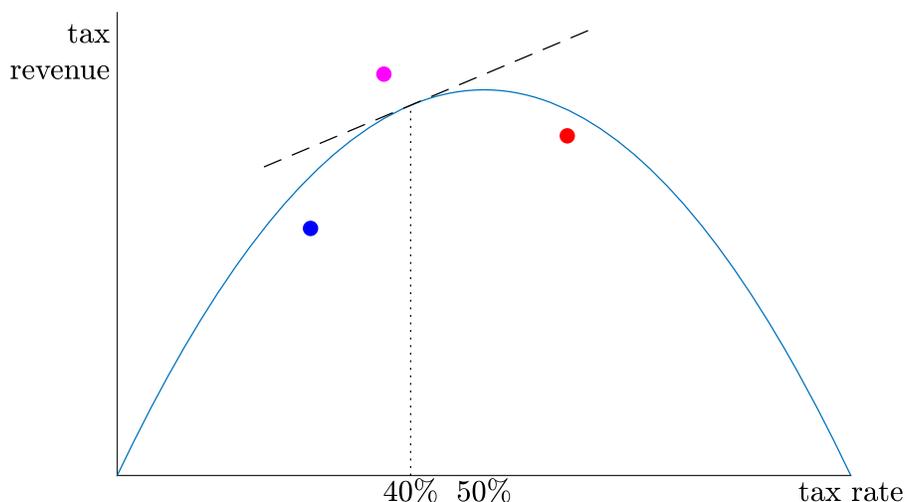


FIGURE 2: The mean tax rate is 40%. Evaluating the marginal effect at this mean value gives the slope shown with the dashed line.

The general definition of the **marginal effect at the mean (MEM)** of regressor  $x_j$  on  $y$  (when there are no interaction terms) is

$$\text{MEM} \equiv \frac{\partial \hat{y}}{\partial x_j}(\bar{x}_j), \quad (5)$$

where  $\bar{x}_j$  is the mean of regressor  $x_j$ .

## 4 Interaction Terms

Things can get a bit messier when dealing with interaction terms. For instance, you might have something like

$$\hat{y} = b_1 + b_2x + b_3x^2 + \beta_4xz,$$

which has marginal effect

$$\frac{\partial \hat{y}}{\partial x} = b_2 + 2b_3x + \beta_4z.$$

In this case, you would have to use data for both  $x$  and  $z$  to calculate the marginal effect or any of its summaries.

For example, if you want to calculate the MEM, you'd plug in both the mean of  $x$  and the mean of  $z$ , giving

$$\text{MEM} = b_2 + 2b_3\bar{x} + \beta_4\bar{z}.$$

In other words, we're evaluating the marginal effect at the mean of regressor  $x$  and the

mean of regressor  $z$ .

Likewise, the AME would be

$$\text{AME} = \frac{1}{n} \sum_{i=1}^n (b_2 + 2b_3x_i + \beta_4z_i),$$

where the AME for each observation  $i$  requires plugging in both  $x_i$  and  $z_i$  to evaluate.

This is here mainly to let you know that in a multivariable regression with interaction terms, marginal effect will depend on other regressors.