Problem 1. The state of California is evil and limits the number of permits they will issue to ice cream truck vendors to 400 permits. The inverse demand for ice cream truck permits is $P = 10,000 - 10Q$, and the marginal cost of a permit is $\text{MC} = 1,000$ because California's bureaucracy is out of control.

(a) What would be the welfare-maximizing quantity of permits to issue?

Solution. Welfare is maximized when deadweight loss is minimized, and there is zero deadweight loss when $P = \text{MC}$. In this case, $P^* = 1,000$ and $Q^* = 900$.

(b) If the state of California wanted to sell the permits and maximize revenue, how many would they sell?

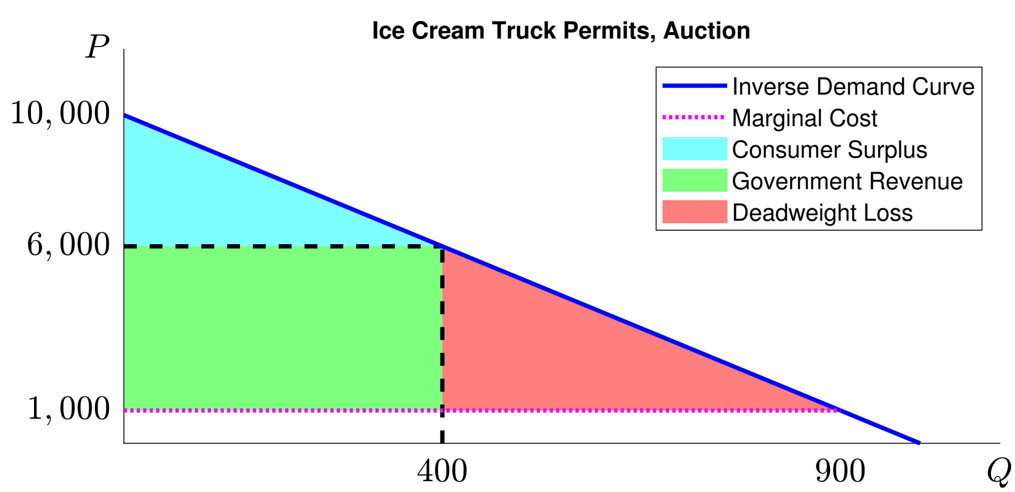
Solution. To maximize revenue, the state of California will behave like a monopolist. Its marginal revenue is $\text{MR} = 10,000 - 20Q := 1,000$, which gives $Q^* = 450$ sold at a price of $P^* = 5,500$.

(c) Suppose the state of California auctions 400 permits. Calculate consumer surplus, government revenue, and deadweight loss.

Solution. If there is an auction, then only those who value the permit the most will actually pay for, and therefore receive, a permit. Therefore we can simply look at $Q^* = 400$, which gives $P^* = 6,000$. Above price and below demand is consumer surplus; below price and above marginal cost is government revenue; and the remainder under demand is deadweight loss.

$$\begin{aligned} \text{CS} &= 0.5(10,000 - 6,000)400 &&= 800,000, \\ \text{GR} &= (6,000 - 1,000)400 &&= 2,000,000, \\ \text{DWL} &= 0.5(6,000 - 1,000)(900 - 400) &&= 1,250,000. \end{aligned}$$

Therefore total welfare is $2,800,000$. Deadweight loss comes from allocative inefficiency: there are simply too few permits relative to the socially optimal number.

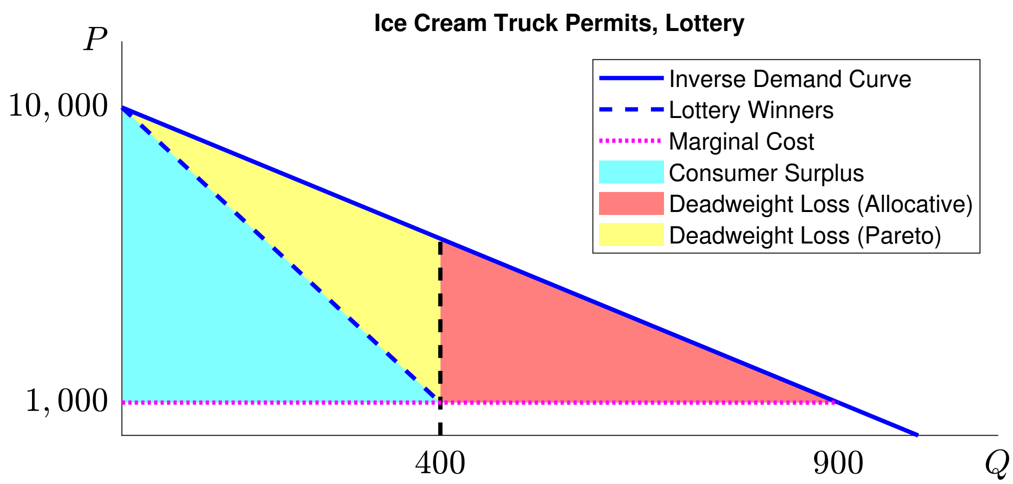


(d) As an alternative, the state of California considers a lottery in which all interested ice cream vendors—those who would have purchased a permit without the government limitation as in part (a)—would apply for a permit. The 400 permits would be randomly allocated to ice cream vendors, which cannot be traded. Relative to part (c), how would consumer surplus and welfare change?

Solution. The state of California will allow any of the 900 ice cream vendors with a valuation no less than marginal cost to enter the lottery, but only 400 of those ice cream vendors will actually be given a permit. The important insight is that the lottery does not necessarily give the highest-valuation ice cream vendors the permit (it's random), which creates Pareto inefficiency in addition to allocative inefficiency.

Because only a random 4/9ths of the tenable ice cream vendors will actually obtain a permit, the lottery winners can be visualized by rotating the inverse demand curve inwards (i.e. quantity shrinks).

$$\begin{aligned} \text{CS} &= 0.5(10{,}000 - 1{,}000)400 &&= 1{,}800{,}000, \\ \text{DWL} &= 0.5(10{,}000 - 1{,}000)(900 - 400) &&= 2{,}250{,}000. \end{aligned}$$



Consumer surplus has increased by one million, but total welfare has fallen by one million. The loss in total welfare is due to the fact that some people who value the permit highly do not actually receive it since the lottery randomizes who actually receives it (i.e. Pareto inefficiency). If resale were possible, then the low-valuation lottery winners would just sell the permit to a higher-valuation lottery loser and we'd end up with welfare equivalent to part (c).

Problem 2. Two firms compete in a Bertrand market to produce a good. Inverse demand is given by $P = 120 - Q$, and the marginal cost of each firm is given by $\text{MC} = 40$.

(a) How much of the good is produced by each firm in (unregulated) equilibrium?

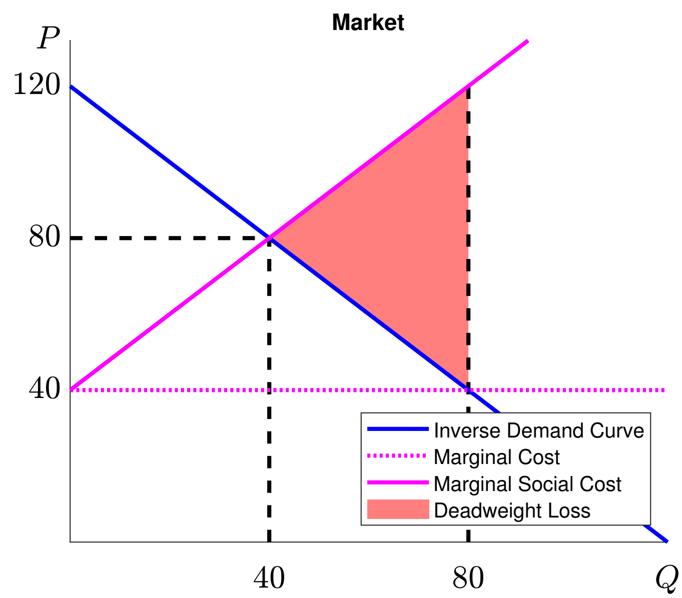
Solution. Bertrand implies $P^* = \text{MC} = 40$. Then from $40 = 120 - Q$, we get $Q^* = 80$. So each firm produces 40 individually.

(b) The production of the good generates an externality that depends on the amount produced. The marginal external costs (i.e. the externality) is given by $\text{MEC} = Q$, so the marginal social costs for the industry are $\text{MSC} = \text{MC} + \text{MEC} = 40 + Q$. What is the socially optimal price and quantity?

Solution. The socially-optimal quantity is produced when MSC intersects the demand curve, i.e. where $40 + Q = 120 - Q$, which gives $Q^* = 40$. It follows that the socially-optimal price is $P^* = 120 - 40 = 80$.

(c) Given your answers to part (b), what is the deadweight loss of the unregulated market in part (a)?

Solution. We've determined from part (b) that the socially-optimal production is $Q = 40$. When unregulated as in part (a), production is $Q = 80$, that is, too much is produced. The deadweight loss from overproduction is the area below MSC and above inverse demand from $Q = 40$ up to $Q = 80$ (i.e. the interval of overproduction), calculated to be $\text{DWL} = 0.5(120 - 40)(80 - 40) = 1600$.



(d) What is the optimal Pigouvian tax to charge the two Bertrand competitors?

Solution. Without any tax, Bertrand firms produce a total of $Q = 80$ at a price of $P = 40$. We want them to produce $Q = 40$ at a price of $P = 80$. Because the socially-optimal price is 40 higher, that means we need a Pigouvian tax of $t = 40$ per unit produced, which increases the (private) marginal cost per unit up to 80.

(e) Would your answer to part (d) change if the two firms were instead a monopoly? If not, briefly explain why not. If so, what is the optimal tax?

Solution. A monopoly sets MR = MC. Total revenue is TR $= 120Q - Q^2$, so MR $= 120 - 2Q$. Therefore the monopoly sets $120 - 2Q = 40$, which yields $Q^* = 40$ and $P^* = 120 - 40 = 80$. Oh look, the monopoly is already producing at the socially-optimal quantity. The negative externality suggests overproduction; the monopoly suggests underproduction; and in this case those two forces exactly cancel each other out, no tax needed.

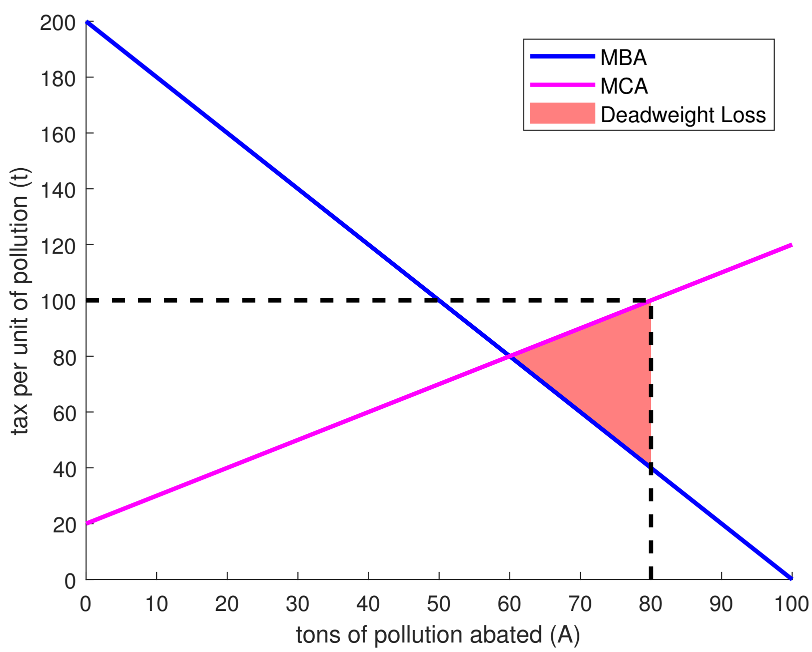
Problem 3. Energy companies deliver natural gas to residential consumers.

(a) The federal government decides to tax air pollution created when natural gas is burned. Assume that the marginal cost of abatement and the marginal benefit of abatement curves are respectively given by MBA $= 200 - 2A$ and MCA $= 20 + A$, where A is abatement measured in tons. What tax should the government set?

Solution. Social welfare is maximized when the marginal benefit of abatement equals the marginal cost of abatement. In this case, when $200 - 2A = 20 + A$, which gives $A^* = 60$. Plug this in for A in either MBA or MCA and you get $t^* = 80$. This says that 60 tons of pollution should be cleaned up at a cost (i.e. tax) of \$80 per ton.

(b) Suppose that instead of the tax you just calculated, the government sets a tax rate of \$100/ton. Draw a diagram illustrating the deadweight loss and calculate the size of deadweight loss.

Solution. At the tax of \$100, too much pollution is being abated relative to the socially-optimal level: the cost exceeds the benefit, so there is deadweight loss. The DWL triangle has area DWL $= 0.5(100 - 40)(80 - 60) = 600$.



(c) Suppose the company emits 200 tons of pollution before any policy is put into place. How many tons of pollution should be permitted to maximize social welfare? What will be the market price of a permit to emit one ton of pollution? Which energy companies will buy the permits?

Solution. 200 tons of pollution were emitted before any policy, and we want to abate 60 tons of pollution, so $200 - 60 = 140$ tons of pollution should be permitted. This means issuing 140 permits, each permit allowing one ton of pollution, for \$80 each. Energy companies with an MCA lower than \$80 will choose to abate; whereas energy companies with an MCA higher than \$80 will purchase the permit and pollute. (Or technologies with an MCA lower than \$80 are cheaper to abate with; whereas technologies with an MCA higher than \$80 are cheaper to pollute with via permit.)