Sets and Bayes Rule

Properties of Sets

$$P(A) = P(A|B)P(B) + P(A|\neg B)P(\neg B)$$

$$P(\neg A) = 1 - P(A)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A|B)P(B) = P(B|A)P(A)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Bayes Rule

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|\neg A)P(\neg A)}$$

Measures of Spread

Variance

$$\mathrm{Var}(X) \equiv E\left[(X - E[X])^2\right]$$

- $\mathrm{Var}(c) = 0$, where c is a constant
- $\mathrm{Var}(X) = E[X^2] - E[X]^2$
- $\mathrm{Var}(aX) = a^2 \cdot \mathrm{Var}(X)$, where a is a constant
- $\mathrm{Var}(X+Y) = \mathrm{Var}(X) + \mathrm{Var}(Y) + 2\,\mathrm{Cov}(X,Y)$
- $\mathrm{Var}(X+Y) = \mathrm{Var}(X) + \mathrm{Var}(Y)$, independent X, Y

Covariance

$$\mathrm{Cov}(X,Y) \equiv E\left[(X - E[X])(Y - E[y])\right]$$

- $\mathrm{Cov}(X,X) = \mathrm{Var}(X)$
- $\mathrm{Cov}(X,c) = 0$, where c is a constant
- $\mathrm{Cov}(X,Y) = E[XY] - E[X]E[Y]$
- $\mathrm{Cov}(X,Y) = 0$ for independent X, Y
- $\mathrm{Cov}(X,Y) = \mathrm{Cov}(Y,X)$
- $\mathrm{Cov}(aX,Y) = a \cdot \mathrm{Cov}(X,Y)$
- $\mathrm{Cov}(X,Y+Z) = \mathrm{Cov}(X,Y) + \mathrm{Cov}(X,Z)$

Correlation

$$\mathrm{Corr}(X,Y) \equiv \frac{\mathrm{Cov}(X,Y)}{\sqrt{\mathrm{Var}(X)\,\mathrm{Var}(Y)}}$$

- $\mathrm{Corr}(X,X) = 1$
- $\mathrm{Corr}(X,Y) = \mathrm{Corr}(Y,X)$
- X, Y independent $\implies \mathrm{Corr}(X,Y) = 0$
- $-1 \leq \mathrm{Corr}(X,Y) \leq 1$
- $\mathrm{Corr}(aX,Y) = \mathrm{sgn}(a)\,\mathrm{Corr}(X,Y)$

Joint, Marginal, Conditional

Conditional Expectation

$$E[X|Y=y] = \sum_{\text{all } x} xP(X=x|Y=y)$$

$$E[X|Y=y] = \int_{\text{over } x} x f_{X|Y}(x,y)\,dx$$

$$f_Y(y) = \int_{\text{over } x} f(x,y)\,dx$$

- $E[E(Y|X)] = E[Y]$
- $E[X+Y|Z] = E[X|Z] + E[Y|Z]$
- $E[Y] = \sum_{\text{all } x} E[Y|X=x]P(X=x)$
- $E[g(X)Y|X] = g(X)E[Y|X]$
- $\mathrm{Var}(X) = E[\mathrm{Var}(X|Y)] + \mathrm{Var}(E[X|Y])$

Joint, Marginal, Conditional Probabilities

$$P(X=x, Y=y) = P(x,y)$$

$$P(X=x) = \sum_{\text{all } y} P(X=x, Y=y) = \sum_{\text{all } y} P(X=x|Y=y)P(Y=y)$$

$$f(x,y) = \int_{\text{over } x}\int_{y(x)} f(x,y)\,dxdy = 1$$

$$f_X(x) = \int_{y(x)} f(x,y)\,dy$$

$$f_{X|Y}(x,y) = \frac{f(x,y)}{f_Y(y)}$$

Moment Generating Functions

Definitions and Properties

$$M_X(t) = E\left[e^{tX}\right] = \sum_{\text{all } x} e^{tx} p_X(x)$$

$$M_X(t) = E\left[e^{tX}\right] = \int_{\text{over } x} e^{tx} f_X(x)\,dx$$

$$M_{X+Y}(t) = M_X(t) \cdot M_Y(t)$$

$$\frac{d}{dt}[M_X(t)] = E[X], \quad \frac{d^2}{dt^2}[M_X(t)] = E[X^2]$$

Common Distributions

$$\mathrm{Binomial}(n,p):\ (1 - p + pe^t)^n$$

$$\mathrm{Geometric}(p):\ \frac{pe^t}{1-(1-p)e^t} \text{ for } t < -\ln(1-p)$$

$$\mathrm{Poisson}(\lambda):\ e^{\lambda(e^t - 1)}$$

$$\mathrm{Gamma}(r,\lambda):\ \left(1 - \frac{t}{\lambda}\right)^{-r}$$

$$\mathcal{N}\left(\mu, \sigma^2\right):\ \exp\left(t\mu + \frac{1}{2}\sigma^2 t^2\right)$$